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USING MATHEMATICAL REPRESENTATION TO SOLVE WORD PROBLEM: ATTENTION CONTROL, ORGANIZING INFORMATION AND ELABORATION

[1]Bansu Irianto Ansari

[1]Mathematics Education of Serambi Mekkah University

bansuansari58@gmail.com

ABSTRACT

Mathematical word problems includes problem-solving category. Many students are confused when the teacher gives the word problems because they do not know where to begin their work. The purpose of this study was to determine how students use attention control, organizing information, and elaboration when completing word problem. The sample consisted of 54 high school students from three different school levels were tested twice for two months. The result of the study showed there was progress from the initial test and the final test, which is about the students select and choose the manner deemed appropriate, the organization of information for managing the use of concepts and rules to describes the relationship to writing equations and solve them. Study showed that 1st group study, 2nd group study, and control group tended to have different *mathematical expressions* and *written texts* ability while solving algebra and geometry problems, whereas drawing ability tended to be the same for all 3 groups. Even when all relationships were recognised and correctly symbolised, integrating them into an equation was a common difficulty.

Keywords:*Mathematical Representation; Attention Control; Organizing Information; Elaboration*

INTRODUCTION

Mathematical representations is a skill that needs to be improved on the students to have the skill to solve word problems. NCTM (2000) states the ability of representation is one of the goals of learning mathematics in school. NCTM (1989) states representation is a new forms as a result of the translation of a problem or idea or translation of a diagram or physical models into symbols or words. An example is the visual presentation such as drawing, chart and tables, math expression and writes their own language, both formal and informal (written texts). There are three students thinking strategies when completing tasks those are attention control, organizing information and elaboration.

Kramarski (2000) stated that the students mathematical representations were classified into four categories: (a) verbal arguments based on visual analysis of the chart; (b) verbal arguments based on formal; (c) numeric/algebraic arguments; and (d) arguments based on drawings that students added to the graph. Based on above the point (a) dan (b) relevant with written texs, point (c) relevant with methematical expression, dan point (d) relevant with drawings. There are three aspect mathematical representations will be assessed in the study that isconstruction of conceptual models such as drawings, tables, greaphs and diagrams (*drawing*); Create mathematical models (math expression) and verbal arguments base on analysis of formal images and concepts (*written texts*).Two judges who are expert in mathematics education analyzed students'explanations. Inter judge reliability coefficient was 0.88

Attention control refers to an individual's capacity to choose what they pay attention to and what they ignore also known as endogenous attention or executive attention in lay terms, attention control can be described as an individual's ability to concentrate. The Attention of student will be higher while encounter a question, because the answer need several associated concepts.In the Information organization thinking, students remember learned information in several ways. Information divide according to its categories then arranged and organized into patterns that can be used as problem solving.Elaboration thinking strategy is to associate lessons learned and integrate those to preexistent knowledge, so that students able to change complex information become simple ones as if a new packet knowledge. Then students create hypothesis to solve problems (Gagne, 1985).

According to the explanation described above, features of thinking strategy pattern are:Attention control; (1) Able to remember more concepts that are relevant to problem, (2) Attention control to recall rules associated to problem, (3) Analyzing problem using available information.Information organization; (1) Organize all facts in order to become systematic information, (2) Create patterns and ideas then associate those to problem solving implementation. Elaboration; (1) Associate lessons learned and integrate those to preexistent knowledge; (2) Change complex information become simple information via models; (3) create hypothesis than implement proper problem solving method.

The characteristics of attention control thinking are relevant to write their own language, both formal and informal (written text), Information organization thinking is relevant to visual presentation such as drawing, chart and tables. Elaboration thinking is relevant to arrange model (mathematical expression), as expressed by NCTM (1989).

METHOD

1. Subjects

Subject samples are 54 students of senior high school (class X, age 15-16 years) chosen according to school level and basic knowledge level. 18 students are chosen at each school level and divided into 3 groups (1st group study, 2nd group study, and control group). Each group consists of 6 students. Respectively 2 students which have high initial test score (H), medium initial test score (M), and low initial test score (L) are chosen from each group study. Subject samples are given initial test, treatment, and final test. All classes involved were mixed-ability classes.

2. Test

The test comprised three algebra word problem and three geometry varying difficulty. The final test, students are divided into 2 groups, first group consist of 27 students solving algebra problems and second group consist of 27 student solving geometry problems.

Tabel 1. Subjecs sampel according tofinal test group

School levels/ Final Test	**Group Study 1**	**Group Study 2**	**Control Group**	**Total**
	Initial Test	Initial Test	Initial Test	

	H	M	L	H	M	L	H	M	L	
Hight (H)	2	2	2	2	2	2	2	2	2	18
Middle (M	2	2	2	2	2	2	2	2	2	18
Low (L)	2	2	2	2	2	2	2	2	2	18
Total	6	6	6	6	6	6	6	6	6	54
Algebra test	3	3	3	3	3	3	3	3	3	27
Geometry	3	3	3	3	3	3	3	3	3	27

RESULTS AND DISCUSSION

1. Recapitulation and explanation of final test result

Table 2. Math representativeaccording to group study anditems test

Items Test	**Mathematical Representation**	**Groups**			**Total**
		Study 1	Study 2	Control	
Algebra (1)	Written Texts	24	23	24	**71**
	Drawing	-	-	-	**-**
	Mat. Expres.	9	7	6	**22**
Algebra (2)	Written Texts	-	-	-	**-**
	Drawing	27	27	26	**80**
	Mat. Expres.	12	11	6	**29**
Algebra (3)	Written Texts	23	21	21	**65**
	Drawing	-	-	-	**-**
	Mat. Express	13	10	7	**30**
Geometry (1)	Written Texts		24	22	**72**
	Drawing	26	-	-	**-**
	Mat. Expres.	-	20	17	**57**
		20			
Geometry (2)	Written Texts	-	-	-	**-**
	Drawing		27	24	**78**
	Mat. Expres.	27	5	2	**14**
		7			

Geometry (3)	Written Texts	27	25	25	**77**
	Drawing	21	24	22	**67**
	Mat. Expres.	18	13	10	**41**
Total		**254**	**237**	**212**	**703**

According to table 2, the frequency of using math representative according to group study is as given in table 3.

Tabel 3. Frequency of using representative math according to group

Representasi	**Group**		
	Study 1	Study 2	**Control**
Written Texts	100 (35,1 %)	93 (32,6 %)	**92 (32,3 %)**
Drawing	75 (33,3 %)	78 (34,7 %)	**72 (32,0 %)**
Math. Exp.	79 (40,9 %)	66 (34,2 %)	**48 (24,9 %)**
Jumlah	**254 (36,1 %)**	**237 (33,7 %)**	**212 (30,2 %)**

Percentage of using representative aspect at each group can be seen as group ability of those aspects.For example, percentage of experiment group 1st using *written texts*, *drawing*, and *mathematical expressions* is 35,1%, 33,3% dan 40,9%, so that math representative ability of this group is 35, 33,3 dan 40,9 respectively.Experiment result show that 1st group study, 2nd group study, and control group tend to have different *mathematical expressions* and *written texts* ability while solving algebra and geometry problems, whereas drawing ability tend to same for all 3 groups.

This experiment result is relevant to previous invention by Esty&Teppo (1996:47). They stated that student able to solve problem given in form of narrative drawing but student failed to build a problem solving formula. In the table 3, can be seen that amount of frequency using representative forms in problem solving among all 3 groups is not highly varying, but conceptually 1st groups conducted better consecutive cognitive process which is attaining relatively better result.

According to experiment on 54 students, the 1st group is prominent in *written texts* and *mathematical expressions* aspect, whereas 2nd group is prominent in *drawing* aspect. The result shows that representative that is treated using TTW strategy give positive effect to student in the final test.

According to student answers, can be analyzed that, there are 3 cognitive processes conducted by students in case of selecting and choosing the rules in order to solve algebra and geometry problems. The students use drawing as model, formulating math equation (associating ideas), describing in own words (written word), and combining all those aspects. In this case, cognitive process is student math representative.

Drawing is more frequent used by 2nd group study, whereas formulating math model and explaining based on student own expression (written word) are more frequent used by 1st group study, and combination of drawing, formulating math model, and own expression are more frequent used by 1st group study,

In other side, students of control group are more prominent used explaining based on student own expression than another two aspects representative.Drawing and model created by 1st group and 2nd group are more relevant than control group. Relevant drawing and model are first from student which has high initial knowledge.Drawing and math model arranged by both group study are combination of information contained in problems and “package of knowledge” about algebra and geometry that are previously learned.“Package of knowledge” is related geometry rules that are used properly. Based on drawing, student forecast the next steps including selecting rules correspond to drawing. Process conducted by students who have high initial knowledge is also conducted by studentswho have medium and low initial knowledge. Because of the limitation “packet knowledge”, student did not understand the associated concept in the problems, so that drawing sketched are not predictive with the result that can’t be used to arrange further step and at last students are difficult in selecting the rules to attain good result. These are conformable to Gagne (1985) that state, students do not understand the associated concept because they are not save information in meaningful proposition, instead in open proposition.

2. Equations as Descriptions of Procedures Used for Calculating

Several students calculated answers to each problem by arithmetic reasoning, and then tried to represent these calculations as equations. This method of dealing with algebra word problems has been observed by other researchers (MacGregor & Stacey, 1996). These equations were not representations of problem structure, but descriptions of the procedure used to calculate a value for one of the unknowns. For example, a student wrote the equation for Problem 2 as,

$$x = (87-18) \div 3$$

and others wrote

$$x = 87-18 = 63 \div 3 = 23$$

wherestands for the time taken to wash the first car. It can be argued that technically, the first of these is an acceptable equation. However in the harder problems on the test, which were too difficult to solve by mental reasoning and arithmetic, students who were limited to writing a description of the solution method had no chance of success. Lacking the support of an algebraic representation of the problem, they were unable to devise a solution.

Figure 2. Attempts to integrate problem information

(i)	Sat	Sun	Mon	(ii)	X
	$x \rightarrow$	$x + 7 \rightarrow$	$x + 13 = 80$		X
					$x + 7$
					$\underline{x + 13}$
					60

(iii) 1 = y

2 = y +7

3 = y + 13

1 + 2 + 3 = 80

(iv) $x, x +7, x +13$, 80

(v) ◄- - - - - - - - - - 80 - - - - - - - - - - ►

α = ? b = α +7 c = α +13

Our data indicate that major difficulties in formulating equations in the test did not lie in students' failure to comprehend the written information, to understand the problem structure, or to see how the parts were related to each other and to the whole. Most students could solve the problems by non-algebraic method, providing confirmation that understanding the problem situation was not a difficulty. For the students who tried to use algebra, the main obstacles to success were (a) incorrect use of algebraic syntax, and (b) failure to integrate the given information as an equation or set of equation. There were others who wrote correct equations that could be used to solve the problems, but did not use there equations, apparently not knowing how to use the notation as a tool for deductive reasoning.

Moreover, our data indicate that, for most students in the sample, naming variables and understanding relations were not difficult for the simple problems we used. Most students who tried algebra could name quantities, and there was little difficulty related to expressing several quantities in terms of one variable. However there were several instances of students who named the three parts in a problem appropriately (e.g., $x + x + 7,\ x + 13$) but did not try to relate them to the total. As we have shown, some students used unconventional formats such as arrow-diagrams, vertical addition, or invented notations to try to denote the idea that the sum of the parts is equal to the total given. They were unable to write an equation to express the structure of the problem situation. Others had not learned that an equation is written to represent the problem situation; they wrote a description of the calculation procedure they had used to solve the problem. These students have perceived the equations as a formula for calculating. They need to know that algebra can also be used to extend and support logical reasoning; its purpose in problem-solving is not to describe a solution procedure that has already been constructed mentally.

In a typical school algebra curriculum, the first problems given to students to solve by algebraic means can also be solved by simple arithmetic, intuitive reasoning, or a simple guess-and-check. Until they achieve a certain level of fluency, students see algebra as an extra difficulty or unnecessry task imposed by teachers for no obvious purpose and not as a useful tool for making problem-solving simpler. This attitude is reasonable, since the problems they have so far encountered (such as the three problems presented in this paper) are not good examples of the power of algebra. We support this view, while reminding readers that it is difficult to find problems that are sufficiently complex to warrant an algebraic solution but easy enough for students to work through with understanding and lear from.It is generally assumed that when comprehending a mathematical problem and preparing to solve it people construct a mental representation of some kind. Several theories have been proposed about the form and function of these representations. Using the context of elementary-grade arithmetic problems.Kintsch and Greeno (1985) proposed that a problem representation is built in several steps, beginning with a conceptual representation of meaning in the form of a set of propositions (called a *conceptual text base* by Kintsch and Greeno). The individual combines this propositional representation with other general and specific knowledge to construct the second representation - an integrated and articulated mental model of the problem situation. We suggest that the initial set of propositions (Johnson-Laird's *propositional representation*) is sufficient for solving problems by the guess-and-check method. The proposional representation provides comprehension of each piece of information given without necessarily relating it other information in a single model. The problem - solver guesses a value for unknown, and checks whether it allows each proposition to be true. The process is essentially substitution in expressions. Almost all the problem we have seen in textbooks for the first four years of school algebra can be solved quickly by this method because of the nature of the numbers involved, and consequently many students use it. Students who have a good number sense often reach the answer after three or four guesses. An algebraic equation does not represent separate pieces of propositional information but arises from the integrated metal model that is produced in the later stage of comprehension. As we have seen, some students do not know how to represent this model as an equation.

In their investigation of algebra word-problem comprehension, Nathan, Kintsch and Greeno (1992) have pointed out that students may understand a problem in everyday terms but be unable to represent its formal aspect as required for an algebraic solution. These researchers suggest that features of the student's cognitive representation of the problem determine what information is available for reasoning. Our data provide no evidence that students' mental models were inadequate or incorrect. The students demonstrated their capabilities in comprehension, logical reasoning, written calculation, mental arithmetic, and problem-solving by non-algebraic methods. There is no obvious explanation for their difficulties in constructing equations for the simple problem they were given. It seems likely that they had not had sufficient experience.

CONCLUSION

We conclude that formulating an equation is not an intuitive way to represent a problem, but needs to be carefully taught. Reluctance to formulaic an equation may be one indicator of the gap between arithmetic and algebraic thinking that is now widely recognized in the literature. For them, an equation is an abbreviated way of writing the terms of the problem: a summary". Our data support this assertion. In certain contexts students are familiar with equations as formulas, that is, as instructions for arithmetic calculation. They know how to use a formula by substituting values and calculating the answer. The concept of an equation as a statement about relationships, rather than as a formula, may be crucial to students' ability to use algebra for solving problems. Student's perceptions of what an equation represents and how it relates to a problem are being investigated in the current stage of our project.

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