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Invited Section Lectures
Invited Panels

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About this book

This book contains abstracts of plenary and invited lectures of the SEOUL ICM 2014. We thank the authors for contributing their abstracts. Twenty plenary lecture abstracts, 179 invited lecture abstracts, two panel discussion abstracts, the Abel Prize lecture abstract and the Emmy Noether lecture abstract are included in this book. Invited lecture abstracts are listed in an alphabetical order by the last name of the first authors within each section. In case of joint authorship, the authors are listed in the order submitted with the presenting author indicated with a "*"

Presentation Code

Abstract Title

IL4.3

Quasimap theory

Ionuţ Ciocan-Fontanine[1,a] and Bumsig Kim[2,b*]

indicates Presenter

[1]University of Minnesota, United States of America
[2]Korea Institute for Advanced Study, Republic of Korea
[a]ciocan@math.umn.edu
[b]bumsig@kias.re.kr

2010 *Mathematics Subject Classification.* 14D20, 14D23, 14N35

Keywords. GIT quotients, Quasimaps, Gromov-Witten theory, Mirror symmetry, Gauged linear σ-models

We provide a short introduction to the theory of ε-stable quasimaps and its applications via wall-crossing to Gromov-Witten theory of GIT targets.

Contents

Plenary Lectures

PL-1

Virtual properties of 3-manifolds

Ian Agol
University of California, Berkeley, United States of America
ianagol@math.berkeley.edu

2010 *Mathematics Subject Classification.* 57M

Keywords. Hyperbolic, 3-manifold

We will discuss the proof of Waldhausen's conjecture that compact aspherical 3-manifolds are virtually Haken, as well as Thurston's conjecture that hyperbolic 3-manifolds are virtually fibered. The proofs depend on major developments in 3-manifold topology of the past decades, including Perelman's resolution of the geometrization conjecture, results of Kahn and Markovic on the existence of immersed surfaces in hyperbolic 3-manifolds, and Gabai's sutured manifold theory. In fact, we prove a more general theorem in geometric group theory concerning hyperbolic groups acting on CAT(0) cube complexes, concepts introduced by Gromov. We resolve a conjecture of Dani Wise about these groups, making use of the theory that Wise developed with collaborators including Bergeron, Haglund, Hsu, and Sageev as well as the theory of relatively hyperbolic Dehn filling developed by Groves-Manning and Osin.

PL-4

L-functions and automorphic representations

James Arthur
University of Toronto, Canada
arthur@math.toronto.edu

2010 *Mathematics Subject Classification.* 11F66, 11F70, 11R37, 11F57, 22F55

Keywords. L-functions, automorphic representations, functoriality, classical groups, discrete spectrum

Our goal is to formulate a theorem that is part of a recent classification of automorphic representations of orthogonal and symplectic groups. To place it in perspective, we devote much of the paper to a historical introduction to the Langlands program. In our attempt to make the article accessible to a general mathematical audience, we have centred it around the theory of L-functions, and its implicit foundation, Langlands' principle of functoriality.

PL-9

Rational points on elliptic and hyperelliptic curves

Manjul Bhargava
Princeton University, United States of America
bhargava@math.princeton.edu

2010 *Mathematics Subject Classification.* 11G05, 11G30, 11R45, 14H25, 20G30

Keywords. Elliptic curve, rank, hyperelliptc curve, rational points, Birch–Swinnerton-Dyer Conjecture

A *hyperelliptic curve* C over $\mathbb{Q}$ is the graph of an equation of the form $y^2 = f(x)$, where f is

a polynomial with coefficients in the rational numbers $\mathbb{Q}$. The special case where the degree of f is 3 is called an *elliptic curve* E over $\mathbb{Q}$ which, as we will discuss, has many special properties not shared by general hyperelliptic curves C. A solution (x, y) to $C : y^2 = f(x)$, with x and y rational numbers, is called a *rational point* on C.

Given a random elliptic or hyperelliptic curve $C : y^2 = f(x)$ over $\mathbb{Q}$ with $f(x)$ of a given degree n, how many rational points do we expect the curve C to have? Equivalently, how often do we expect a random polynomial $f(x)$ of degree n to take a square value over the rational numbers? In this article, we give an overview of a number of recent conjectures and theorems giving some answers and partial answers to this question.

PL-10

Integrable probability

Alexei Borodin
Massachusetts Institute of Technology, United States of America
borodin@math.mit.edu

2010 *Mathematics Subject Classification.* 60K35, 82B23, 82C41
Keywords. Integrable, Probability

The goal of the lecture is to survey the emerging field of integrable probability which aims at identifying and analyzing exactly solvable probabilistic models. The models and results are often easy to describe, yet difficult to find, and they carry essential information about broad universality classes of stochastic processes. The methods of analysis are largely algebraic, and they are deeply rooted in representation theory.

PL-8

The great beauty of VEM's

Franco Brezzi
Istituto Universitario di Studi Superiori, Pavia, Italy
brezzi@imati.cnr.it

2010 *Mathematics Subject Classification.* 65Nxx, 65N30
Keywords. Virtual Element Methods, Polygonal decompositions, Patch test.

In this paper I review the main features of the (newborn) Virtual Element Method, and of its application to the approximation of boundary value problems for Partial Differential Equations of particular relevance for applications. I will mostly concentrate on the definition of the Virtual Element spaces, that, roughly, consist of (vector valued) functions that are solution of (systems of) partial differential equations in each subdomain of a decomposition of the computational domain into polygons or polyhedra of quite general shape. Then I will give some hint on the use of these spaces for the discretization of some classical toy-problems like Heat conduction, Darcy flows, and Magnetostatic problems.

PL-3

Mathematics of sparsity (and a few other things)

Emmanuel J. Candès
Stanford University, United States of America
candes@stanford.edu

2010 *Mathematics Subject Classification.* 00A69

Keywords. Underdetermined systems of linear equations, compressive sensing, matrix completion, sparsity, low-rank-matrices, ℓ_1 norm, nuclear norm, convex programing, Gaussian widths

In the last decade, there has been considerable interest in understanding when it is possible to find structured solutions to underdetermined systems of linear equations. This paper surveys some of the mathematical theories, known as compressive sensing and matrix completion, that have been developed to find sparse and low-rank solutions via convex programming techniques. Our exposition emphasizes the important role of the concept of incoherence.

PL-5

Hyperbolic P.D.E. and Lorentzian Geometry

Demetrios Christodoulou
ETH-Zurich, Switzerland
demetri@math.ethz.ch

2010 *Mathematics Subject Classification.* 35L72, 53C50, 83C57, 35L67, 76L05

Keywords. Hyperbolic partial differential equations, Lorentzian geometry, general relativity, fluid mechanics

Recent developments are discussed which deepen our understanding of the relationship between hyperbolic p.d.e. and Lorentzian geometry. These developments are connected with progress in the analysis of the Einstein equations of general relativity and in the analysis of the Euler equations of the mechanics of compressible fluids.

PL-12

Minimal surfaces - variational theory and applications

Fernando Codá Marques
IMPA, Brazil
coda@impa.br

2010 *Mathematics Subject Classification.* 53C42, 49Q05

Keywords. Minimal surfaces, calculus of variations, conformal geometry, three-manifold topology

Minimal surfaces are among the most natural objects in Differential Geometry, and have been studied for the past 250 years ever since the pioneering work of Lagrange. The subject is characterized by a profound beauty, but perhaps even more remarkably, minimal surfaces (or minimal submanifolds) have encountered striking applications in other fields, like three-dimensional topology, mathematical physics, conformal geometry, among others. Even though it has been the subject of intense activity, many basic open problems still remain. In

this lecture we will survey recent advances in this area and discuss some future directions. We will give special emphasis to the variational aspects of the theory as well as to the applications to other fields.

PL-16

Random Structures and Algorithms

Alan Frieze
Carnegie Mellon University, United States of America
alan@random.math.cmu.edu

2010 *Mathematics Subject Classification.* 05C80, 68Q25

Keywords. Random Graphs, Probabilitic Analysis of Algorithms

We provide an introduction to the analysis of random combinatorial structures and some of the associated computational problems.

PL-14

Approximate algebraic structure

Ben Green
University of Oxford, United Kingdom
ben.green@maths.ox.ac.uk

2010 *Mathematics Subject Classification.* 11B30

Keywords. Approximate group, Gowers norm, nilsequence, additive combinatorics, arithmetic combinatorics

We discuss a selection of recent developments in arithmetic combinatorics having to do with "approximate algebraic structure" together with some of their applications.

PL-2

Mori geometry meets Cartan geometry: Varieties of minimal rational tangents

Jun-Muk Hwang
KIAS, Republic of Korea
jmhwang@kias.re.kr

2010 *Mathematics Subject Classification.* 14J40, 53B99, 14J45

Keywords. Varieties of minimal rational tangents, uniruled projective manifolds, Cartan geometry, G-structures

We give an introduction to the theory of varieties of minimal rational tangents, emphasizing its aspect as a fusion of algebraic geometry and differential geometry, more specifically, a fusion of Mori geometry of minimal rational curves and Cartan geometry of cone structures.

PL-6

The structure of algebraic varieties

János Kollár
Princeton University, United States of America
kollar@math.princeton.edu

2010 *Mathematics Subject Classification.* 14-02, 14E30, 14B05, 14D20

Keywords. Algebraic variety, Mori program, moduli questions

The aim of this address is to give an overview of the main questions and results of the structure theory of higher dimensional algebraic varieties.

PL-11

Random geometry on the sphere

Jean-François Le Gall
Université Paris-Sud, France
jean-francois.legall@math.u-psud.fr

2010 *Mathematics Subject Classification.* 05C80, 60D05, 05C12, 60F17

Keywords. Planar map, triangulation, Brownian map, Gromov-Hausdorff convergence, graph distance

We introduce and study a universal model of random geometry in two dimensions. To this end, we start from a discrete graph drawn on the sphere, which is chosen uniformly at random in a certain class of graphs with a given size n, for instance the class of all triangulations of the sphere with n faces. We equip the vertex set of the graph with the usual graph distance rescaled by the factor $n^{-1/4}$. We then prove that the resulting random metric space converges in distribution as $n \to \infty$, in the Gromov-Hausdorff sense, toward a limiting random compact metric space called the Brownian map, which is universal in the sense that it does not depend on the class of graphs chosen initially. The Brownian map is homeomorphic to the sphere, but its Hausdorff dimension is equal to 4. We obtain detailed information about the structure of geodesics in the Brownian map. We also present the infinite-volume variant of the Brownian map called the Brownian plane, which arises as the scaling limit of the uniform infinite planar quadrangulation. Finally, we discuss certain open problems. This study is motivated in part by the use of random geometry in the physical theory of two-dimensional quantum gravity.

PL-13

Analytic Low-Dimensional Dynamics: from dimension one to two

Mikhail Lyubich
Stony Brook University, United States of America
mlyubich@math.sunysb.edu

2010 *Mathematics Subject Classification.*

Keywords. Hyperbolicity, structural stability, attractor, renormalization, homoclinic tangency, Julia set, Henon map, a priori bounds

Let $f : M \to M$ be an analytic (real or complex) self-map of a manifold, and let f^n stand for

its n-fold iterate. The theory of Analytic Dynamical Systems with discrete time is concerned with understanding the asymptotic behavior of orbits $(f^n x)$. The main goal, as it was articulated in the second half of 20th century, is to describe, in probabilistic terms, the asymptotic distribution of typical orbits for typical systems. This goal is now achieved for unimodal one-dimensional maps, a great progress has been made in complex one-dimensional case, and a transition to the dissipative two-dimensional situation, real and complex, is underway. Renormalization ideas played a crucial role in this story. We will describe all these developments in their interplay.

PL-18

Asymptotics for critical nonlinear dispersive equations

Frank Merle
Universite de Cergy-Pontoise et Institut des Hautes Etudes Scientifiques, France
merle@ihes.fr

2010 *Mathematics Subject Classification.* 35B40, 35B44, 35B33, 35Q53, 35Q55

Keywords. Dispersive nonlinear P.D.E., Criticality, Asymptotics, Blow-up, Soliton

We consider various examples of critical nonlinear partial differential equations which have the following common features: they are Hamiltonian, of dispersive nature, have a conservation law invariant by scaling, and have solutions of nonlinear type (their asymptotic behavior in time differs from the behavior of solutions of linear equations). The main questions concern the possible behaviors one can expect asymptotically in time. Are there many possibilities, or on the contrary very few universal behaviors depending on the type of initial data?

We shall see that the asymptotic behavior of solutions starting with general or constrained initial data is related to very few special solutions of the equation. This will be illustrated through different examples related to classical problems.

For a given equation, the first challenge is to construct solutions with a given behavior, including solutions with interactions between different types of waves (localized/localized or localized/non-localized) leading to nonlinear behavior or blow-up. In many of these problems, a formal guess is made based on a better understanding of the hidden laws of interaction between these waves. Then, from this guess, the questions are how to construct such examples, and why other behaviors in different regimes cannot appear. In particular, these questions are related to finding irreversibility in Hamiltonian systems, and to why oscillations of the solution can be controlled in time. We will see that universality is deeply related to stability or instability of the blow-up regime and the asymptotic behavior.

PL-17

Wild harmonic bundles and twistor $\mathcal{D}$-modules

Takuro Mochizuki
Kyoto University, Japan
takuro@kurims.kyoto-u.ac.jp

2010 *Mathematics Subject Classification.* 14F10, 32C38, 32G20, 32S40, 53C07

Keywords. Twistor structure, Holonomic $\mathcal{D}$-module, Variation of Hodge structure, Singularity, Stokes structure

The notion of twistor structure is a generalization of that of Hodge structure. Harmonic bundles and twistor $\mathcal{D}$-modules are the counterparts of polarized variations of Hodge structure and Hodge modules in the context of twistor structures. The study on harmonic bundles with wild singularity and twistor $\mathcal{D}$-modules lead us to an interesting interaction between global analysis and algebraic analysis. It has resulted in significant progress in the theory of holonomic $\mathcal{D}$-modules also in the context of irregular singularities We will report on these developments.

PL-15

Some mathematical aspects of tumor growth and therapy

Benoît Perthame
Universite Pierre et Marie Curie, France
benoit.perthame@upmc.fr

2010 *Mathematics Subject Classification.* 35K55, 35B25, 76D27, 92C50, 92D25

Keywords. Tumor growth, Hele-Shaw equation, Free boundary problems, Structured population dynamics, Resistance to therapy

Mathematical models of tumor growth, written as partial differential equations or free boundary problems, are now in the toolbox for predicting the evolution of some cancers, using model based image analysis for example. These models serve not only to predict the evolution of cancers in medical treatments but also to understand the biological and mechanical effects that are involved in the tissue growth, the optimal therapy and, in some cases, in their implication in therapeutic failures.

The models under consideration contain several levels of complexity, both in terms of the biological and mechanical effects, and therefore in their mathematical description. The number of scales, from the molecules, to the cell, to the organ and the entire body, explains partly the complexity of the problem.

This paper focusses on two aspects of the problem which can be described with mathematical models keeping some simplicity. They have been chosen so as to cover mathematical questions which stem from both mechanical laws and biological considerations. I shall first present an asymptotic problem describing some mechanical properties of tumor growth and secondly, models of resistance to therapy and cell adaptation again using asymptotic analysis.

PL-19

O-minimality and Diophantine geometry

Jonathan Pila
University of Oxford, United Kingdom
pila@maths.ox.ac.uk

2010 *Mathematics Subject Classification.* 03C64, 11G18

Keywords. O-minimal structure, André-Oort conjecture, Zilber-Pink conjecture

This lecture is concerned with some recent applications of mathematical logic to Diophantine geometry. More precisely it concerns applications of o-minimality, a branch of model theory which treats tame structures in real geometry, to certain finiteness problems descending from the classical conjecture of Mordell.

PL-20

Quasi-randomness and the regularity method in hypergraphs

Vojtěch Rödl
Emory University, United States of America
rodl@mathcs.emory.edu

2010 *Mathematics Subject Classification.* 05C35, 05C65, 05C80, 05D10
Keywords. Szemerédi's theorem, removal lemma, quasi-randomness, Ramsey theory

The probabilistic method is one of the most successful techniques in combinatorics. It enables one to prove results about deterministic objects by immersing them into specially designed probability spaces. One of the more recent techniques employs the idea of quasi-randomness. A quasi-random object is a deterministic object which shares important properties with "typical" objects of the same kind. Szemerédi's regularity lemma asserts, quite remarkably, that every graph can be decomposed into relatively few subgraphs that are quasi-random. In appropriate situations quasi-randomness enables one to find and to enumerate subgraphs of a given isomorphism type. This approach has led to many applications in extremal combinatorics. We discuss some developments and applications of this method and focus on its extensions to hypergraphs.

PL-21

Finite dimensional representations of algebraic supergroups

Vera Serganova
University of California, Berkeley, United States of America
serganov@math.berkeley.edu

2010 *Mathematics Subject Classification.* 17B10, 20G05
Keywords. Lie superalgebra, tensor category, blocks, Borel–Weil–Bott theorem

We review recent results and methods in finite-dimensional representation theory of Lie superalgebras: analogues of Schur-Weyl duality, connections with Deligne's categories, block theory, associated variety, Borel-Weil-Bott theory and categorification.

Emmy Noether Lecture

SL-1

Connecting the McKay correspondence and Schur-Weyl duality

Georgia Benkart
University of Wisconsin-Madison, United States of America
benkart@math.wisc.edu

2010 *Mathematics Subject Classification.* 14E16, 05E10, 20C05

Keywords. McKay correspondence, Schur-Weyl duality

The McKay correspondence and Schur-Weyl duality have inspired a vast amount of research in mathematics and physics. The McKay correspondence establishes a bijection between the finite subgroups of the special unitary 2-by-2 matrices and the simply laced affine Dynkin diagrams from Lie theory. It has led to the discovery of many other remarkable A-D-E phenomena. Schur-Weyl duality reveals hidden connections between the representation theories of two algebras that centralize one another in their actions on the same space. We merge these two notions and explain how this gives new insights and results. Our approach uses the combinatorics of walks on graphs, the Jones basic construction, and partition algebras.

Abel Lecture

SL-2

Topology through Four Centuries

John Milnor
Stony Brook University, United States of America
jack@math.sunysb.edu

2010 *Mathematics Subject Classification.* 01A55, 01A60, 55-03, 57N65, 57R55
Keywords. Topology, History

The first hints of the subject known as topology appeared in the 18-th century. The field took shape in the 19-th century, made dramatic progress during the 20-th century, and is flourishing in the 21-st. The talk will describe a few selected highlights.

1. Logic and Foundation

IL1.3

Model theory of difference fields and applications to algebraic dynamics

Zoé Chatzidakis
Université Paris Diderot-Paris 7, France
zoe@math.univ-paris-diderot.fr

2010 *Mathematics Subject Classification.* 03C60, 12H10
Keywords. Model theory, difference fields, algebraic dynamics

This short paper describes some applications of model theory to problems in algebraic dynamics.

IL1.2

Logic and operator algebras

Ilijas Farah
York University, Canada
ilijasf@gmail.com

2010 *Mathematics Subject Classification.* 03C20, 03C98, 03E15, 03E75, 46L05
Keywords. Logic of metric structures, Borel reducibility, ultraproducts, Classification of C*-algebras, tracial von Neumann algebras

The most recent wave of applications of logic to operator algebras is a young and rapidly developing field. This is a snapshot of the current state of the art.

IL1.4

Amalgamation functors and homology groups in model theory

John Goodrick[1], Byunghan Kim[2,a*], and Alexei Kolesnikov[3]
[1]University of the Andes, Colombia
[2]Yonsei University, Republic of Korea
[3]Towson University, United States of America
[a]bkim@yonsei.ac.kr

2010 *Mathematics Subject Classification.* 03C45, 55N35
Keywords. Amalgamation functors, homology groups, model theory, groupoids, Hurewicz correspondence

We introduce the concept of an amenable class of functors and define homology groups for such classes. Amenable classes of functors arise naturally in model theory from considering types of independent systems of elements. Basic lemmas for computing these homology groups are established, and we discuss connections with type amalgamation properties.

IL1.1

Definability in non-archimedean geometry

François Loeser
Université Pierre et Marie Curie-Paris 6, France
Francois.Loeser@gmail.com

2010 *Mathematics Subject Classification.* 03C98, 12J10, 14G22, 22E35

Keywords. Non-archimedean geometry, p-adic integration, motivic integration, diophantine geometry, Berkovich spaces

We discuss several situations involving valued fields for which the model-theoretic notion of definability plays a central role. In particular, we consider applications to p-adic integration, diophantine geometry and topology of non-archimedean spaces.

IL1.5

Computability theoretic classifications for classes of structures

Antonio Montalbán
University of California, Berkeley, United States of America
antonio@math.berkeley.edu

2010 *Mathematics Subject Classification.* 03D45, 03C57

Keywords. Sigma small classes, back-and-forth relations, rice relations, low property, effective-bi-interpretability

In this paper, we survey recent work in the study of classes of structures from the viewpoint of computability theory. We consider different ways of classifying classes of structures in terms of their global properties, and see how those affect the structures inside the class. On one extreme, we have the classes that are Σ-small. These are the classes which realize only countably many $\exists$-types, and are characterized by having tame computability theoretic behavior. On the opposite end, we look at various notions of completeness for classes which imply that all possible behaviors occur among their structures. We introduce a new notion of completeness, that of being on top for effective-bi-interpretability, which is stronger and more structurally oriented than the previously proposed notions.

IL1.6

Recent developments in A general approach to finite Ramsey theory : foundational aspects and connections with dynamics

Sławomir Solecki
University of Illinois at Urbana-Champaign, United States of America
ssolecki@math.uiuc.edu

2010 *Mathematics Subject Classification.* 03E15, 05D10, 22F50

Keywords. Ramsey theory, topological dynamics

This will be a talk on foundations of finite Ramsey theory. I will present an abstract approach to finite Ramsey theory that reveals the formal algebraic structure underlying results of that

theory. I will formulate within this approach a pigeonhole principle and a Ramsey condition, and state a theorem that the pigeonhole principle implies the Ramsey condition. I will indicate in what way many concrete Ramsey results become special instances, or iterative instances, of this general theorem. I will also describe the context of recent renewed interest in Ramsey theory, which involves connections with topological dynamics.

2. Algebra

IL2.4

On finite-dimensional Hopf algebras

Nicolás Andruskiewitsch
Universidad Nacional de Cordoba, Argentina
kolinka@gmail.com

2010 *Mathematics Subject Classification.* 16T05, 16T20, 17B37, 16T25, 20G42

Keywords. Hopf algebras, quantum groups, Nichols algebras

This is a survey on the state-of-the-art of the classification of finite-dimensional complex Hopf algebras. This general question is addressed through the consideration of different classes of such Hopf algebras. Pointed Hopf algebras constitute the class best understood; the classification of those with abelian group is expected to be completed soon and there is substantial progress in the non-abelian case.

IL2.3

Excision, descent, and singularity in algebraic K-theory.

Guillermo Cortiñas
Universidad de Buenos Aires, Argentina
gcorti@dm.uba.ar

2010 *Mathematics Subject Classification.* 19D55, 19D50, 19E08

Keywords. Algebraic K-theory, cyclic homology, topological algebras, singular varieties

Algebraic K-theory is a homology theory that behaves very well on sufficiently nice objects such as stable C^*-algebras or smooth algebraic varieties, and very badly in singular situations. This survey explains how to exploit this to detect singularity phenomena using K-theory and cyclic homology.

IL2.2

Applications of the classification of finite simple groups

Robert Guralnick
University of Southern California, United States of America
guralnic@usc.edu

2010 *Mathematics Subject Classification.* 20D05, 20B15, 14H30

Keywords. Simple groups, Primitive permutation groups, applications of simple groups

The classification of finite simple groups is one of the most amazing theorems in mathematics. We will survey some applications of this result in number theory, algebraic geometry and group theory.

IL2.1

Higher representation theory and quantum affine Schur-Weyl duality

Seok-Jin Kang
Seoul National University, Republic of Korea
sjkang@snu.ac.kr

2010 *Mathematics Subject Classification.* 17B37, 16E99

Keywords. 2-representation theory, Schur-Weyl duality, Khovanov-Lauda-Rouquier algebra, quantum group

In this article, we explain the main philosophy of 2-representation theory and quantum affine Schur-Weyl duality. The Khovanov-Lauda-Rouquier algebras play a central role in both themes.

IL2.5

Finitely Generated Groups with Controlled Pro-algebraic Completions

Martin Kassabov
Cornell University, United States of America
kassabov@math.cornell.edu

2010 *Mathematics Subject Classification.* 20F69, 20B30, 20E18, 20G05, 20K25

Keywords. Pro-algebraic completion of groups, representation varieties, dimensions of character varieties

We construct finitely generated groups whose pro-algebraic completion is isomorphic to the product of the pro-algebraic completions of groups like $SL_n(\mathbb{Z})$ and $SL_n(\mathbb{Z}[x])$ for different n-es. This leads to examples of groups where the dimensions of the character varieties grow as any function with growth between linear and quadratic.

IL2.6

Model theory and algebraic geometry in groups, non-standard actions and algorithmic problems

Olga Kharlampovich[1,a] and Alexei Myasnikov[2,b*]
[1]Hunter College, CUNY, United States of America
[2]Stevens Institute of Technology, United States of America
[a]okharlampovich@gmail.com
[b]amiasnikov@gmail.com

2010 *Mathematics Subject Classification.* 20E05, 20A15, 20F67

Keywords. Free group, model theory, group actions

We discuss the modern theory of equations in groups, algebraic geometry and model theory in free and hyperbolic groups, as well as group actions on Λ-trees. One of our main tools is a combinatorial process that combines and generalizes a number of known results and algorithms, such as the Makanin-Razborov process for solving equations in groups, Rauzy-Veech induction in dynamical systems, classification of basic group actions in group theory and

topology, and elimination and parametrization theorems in classical algebraic geometry. The development of algebraic geometry comes together with advances in the theory of fully residually free and fully residually hyperbolic groups, which are coordinate groups of irreducible algebraic varieties. We describe finitely generated groups elementarily equivalent to a free non-abelian group (another classification is given by Sela) and show that the first-order theory of a free or a torsion-free hyperbolic group is decidable (solution to Tarski's problems from 1940's). Furthermore, for such groups we give an algorithm for elimination of quantifiers to boolean combinations of $\forall\exists$-formulas. We also provide a description of definable sets in a torsion-free hyperbolic group (in particular, in a free group) and demonstrate that only cyclic subgroups and the whole group are definable in these groups (this solves Malcev's problem of 1965). In the group actions section we describe all finitely presented groups acting freely on Λ-trees (solution to Alperin's and Bass problem of 1990). At the end we outline some related open problems.

IL2.8

Towards the eigenvalue rigidity of Zariski-dense subgroups

Andrei S. Rapinchuk
University of Virginia, United States of America
asr3x@virginia.edu

2010 *Mathematics Subject Classification.* 20G15, 11E72, 53C35

Keywords. Algebraic groups, Zariski-dense subgroups, locally symmetric spaces

We will discuss the notion of weak commensurability of Zariski-dense subgroups of semi-simple algebraic groups over fields of characteristic zero, which enables one to match in a convenient way the eigenvalues of semi-simple elements of these subgroups. The analysis of weakly commensurable arithmetic groups has led to a resolution of some long-standing problems about isospectral locally symmetric spaces. This work has also initiated a number of questions in the theory of algebraic groups dealing with the characterization of absolutely almost simple simply connected algebraic groups having the same isomorphism classes of maximal tori over the field of definition. The recent results in this direction provide evidence to support a new conjectural form of rigidity for arbitrary Zariski-dense subgroups in absolutely almost simple algebraic groups over fields of characteristic zero based on the eigenvalue information ("eigenvalue rigidity").

IL2.7

Local and global Frobenius splitting

Karen E. Smith
University of Michigan, United States of America
kesmith@umich.edu

2010 *Mathematics Subject Classification.* 13A35

Keywords. Frobenius splitting, F-regularity, tight closure, test ideals, compatible splitting

We survey recent progress in local and global Frobenius splitting, explaining the ideas that unify them, including a new way to look at test ideals.

3. Number Theory

IL3.3

Motivic periods and $\mathbb{P}\backslash\{0, 1, \infty\}$

Francis Brown
CNRS-IHES, France
brown@ihes.fr

2010 *Mathematics Subject Classification.* 11M32, 14C15

Keywords. Projective line minus three points, Mixed Tate motives, Fundamental group, Multiple zeta values, Multiple modular values

This is a review of the theory of the motivic fundamental group of the projective line minus three points, and its relation to multiple zeta values.

IL3.2

Completed cohomology and the p-adic Langlands program

Matthew Emerton
University of Chicago, United States of America
matthewjemerton@gmail.com

2010 *Mathematics Subject Classification.* 11F70, 22D12

Keywords. p-adic Langlands program, p-adic Hodge theory, completed cohomology, Galois representations

We discuss some known and conjectural properties of the completed cohomology of congruence quotients associated to reductive groups over $\mathbb{Q}$. We also discuss the conjectural relationships to local and global Galois representations, and a possible p-adic local Langlands correspondence.

IL3.6

Theta correspondence: recent progress and applications

Wee Teck Gan
National University of Singapore, Singapore
matgwt@nus.edu.sg

2010 *Mathematics Subject Classification.* 11F67, 22E50

Keywords. Theta correspondence, Siegel-Weil formula, local Langlands correspondence, Gross-Prasad conjecture, Shimura-Waldspurger correspondence

We describe some recent progress in the theory of theta correspondence over both local and global fields. We also discuss applications of these recent developments to the local Langlands conjecture, the Gross-Prasad conjecture and the theory of automorphic forms for the metaplectic groups.

IL3.4

Small gaps between primes

D. A. Goldston[1,a], Janos Pintz[2,b*], and Cem Yalcin Yıldırım[3,c]
[1]San Jose State Univeristy, United States of America
[2]Alfréd Rényi Institute of Mathematics, Hungary
[3]Bogazici University, Turkey
[a]daniel.goldston@sjsu.edu
[b]pintz.janos@renyi.mta.hu
[c]cyalciny@gmail.com

2010 *Mathematics Subject Classification.* 11N05, 11N36, 11N35

Keywords. Gaps between primes, Prime numbers, Hardy-Littlewood prime tuples conjecture, Twin primes, Sieves

This paper describes the authors' joint research on small gaps between primes in the last decade and how their methods were developed further independently by Zhang, Maynard, and Tao to prove stunning new results on primes. We now know that there are infinitely many primes differing by at most 252, and that one can find k primes a bounded distance (depending on k) apart infinitely often. These results confirm important special cases of the Hardy-Littlewood prime tuples conjecture.

IL3.5

Automorphic Galois representations and the cohomology of Shimura varieties

Michael Harris
Universite Paris-Diderot, France
harris@math.jussieu.fr

2010 *Mathematics Subject Classification.* 11F80, 11F70, 11G18, 11F67

Keywords. Galois representation, Shimura variety, Special values of L-functions

The first part of this report describes the class of representations of Galois groups of number fields that have been attached to automorphic representations. The construction is based on the program for analyzing cohomology of Shimura varieties developed by Langlands and Kottwitz. Using p-adic methods, the class of Galois representations obtainable in this way can be expanded slightly; the link to cohomology remains indispensable at present. It is often possible to characterize the set of Galois representations that can be attached to automorphic forms, using the modularity lifting methods initiated by Wiles a bit over 20 years ago. The report mentions some applications of results of this kind.

The second part of the report explains some recent results on critical values of automorphic L-functions, emphasizing their relation to the motives whose ℓ-adic realizations were discussed in the first part.

IL3.1

The ternary Goldbach problem

Harald Andrés Helfgott
CNRS/ENS, France
harald.helfgott@gmail.com

2010 *Mathematics Subject Classification.* 11P32

Keywords. Ternary Goldbach problem, analytic number theory, additive problems, prime numbers

The ternary Goldbach conjecture, or three-primes problem, states that every odd number n greater than 5 can be written as the sum of three primes. The conjecture, posed in 1742, remained unsolved until now, in spite of great progress in the twentieth century. In 2013 – following a line of research pioneered and developed by Hardy, Littlewood and Vinogradov, among others – the author proved the conjecture. In this, as in many other additive problems, what is at issue is really the proper usage of the limited information we possess on the distribution of prime numbers. The problem serves as a test and whetting-stone for techniques in analysis and number theory – and also as an incentive to think about the relations between existing techniques with greater clarity. We will go over the main ideas of the proof. The basic approach is based on the circle method, the large sieve and exponential sums. For the purposes of this overview, we will not need to work with explicit constants; however, we will discuss what makes certain strategies and procedures not just effective, but efficient, in the sense of leading to good constants. Still, our focus will be on qualitative improvements.

IL3.9

Some problems in analytic number theory for polynomials over a finite field

Zeev Rudnick
Tel Aviv University, Israel
rudnick@post.tau.ac.il

2010 *Mathematics Subject Classification.*

Keywords. Function fields over a finite field, Chowla's conjecture, the additive divisor problem, primes in short intervals

The lecture explores several problems of analytic number theory in the context of function fields over a finite field, where they can be approached by methods different than those of traditional analytic number theory. The resulting theorems can be used to check existing conjectures over the integers, and to generate new ones. Among the problems discussed are: Counting primes in short intervals and in arithmetic progressions; Chowla's conjecture on the autocorrelation of the Möbius function; and the additive divisor problem.

IL3.8

Perfectoid spaces and their applications

Peter Scholze
Universität Bonn, Germany
scholze@math.uni-bonn.de

2010 *Mathematics Subject Classification.* 14G22, 11F80, 14G20, 14C30, 14L05

Keywords. Perfectoid spaces, Rigid-analytic geometry, Shimura varieties, p-adic Hodge theory, Langlands program

We survey the theory of perfectoid spaces and its applications.

IL3.10

Stabilisation de la partie géométrique de la formule des traces tordue

Jean-Loup Waldspurger
Institut de Mathématiques de Jussieu, France
waldspur@math.jussieu.fr

2010 *Mathematics Subject Classification.* 11, 22

Keywords. Twisted trace formula, twisted endoscopy

We explain what is twisted endoscopy. We give the formulation of the geometric part of the twisted trace formula, following the works of Clozel-Labesse-Langlands and Arthut. We explain his stabilization, which is a work in progress, joint with Moeglin.

IL3.7

Translation invariance, exponential sums, and Waring's problem

Trevor D. Wooley
University of Bristol, United Kingdom
matdw@bristol.ac.uk

2010 *Mathematics Subject Classification.* 11L15, 11P05, 11P55

Keywords. Exponential sums, Waring's problem, Hardy-Littlewood method, Weyl sums, Vinogradov's mean value theorem

We describe mean value estimates for exponential sums of degree exceeding 2 that approach those conjectured to be best possible. The vehicle for this recent progress is the efficient congruencing method, which iteratively exploits the translation invariance of associated systems of Diophantine equations to derive powerful congruence constraints on the underlying variables. There are applications to Weyl sums, the distribution of polynomials modulo 1, and other Diophantine problems such as Waring's problem.

IL3.11

Elementary integration of differentials in families and conjectures of Pink

Umberto Zannier
Scuola Normale Superiore, Italy
u.zannier@sns.it

2010 *Mathematics Subject Classification.* 11G10, 11G50

Keywords. Integration, abelian varieties, torsion points, unlikely intersections, conjecture of Pink

In this short survey paper we shall consider, in particular, indefinite integrals of differentials on algebraic curves, trying to express them in *elementary terms*. This is an old-fashioned issue, for which Liouville gave an explicit criterion that may be considered a primordial example of differential algebra. Before presenting some connections with more recent topics, we shall start with an overview of the classical facts, recalling some criteria for elementary integration and relating this with issues of torsion in abelian varieties.

Then we shall turn to differentials in 1-parameter algebraic families, asking for which values of the parameter we can have an elementary integral. (This had been considered already in the 80s by J. Davenport.) The mentioned torsion issues provide a connection of this with a conjecture of R. Pink in the realm of the so-called *Unlikely Intersections*.

In joint work in collaboration with David Masser (still partly in progress), we have proved finiteness of the set of relevant values, under suitable necessary conditions. Here we shall give a brief account of the whole context, pointing out at the end possible links with other problems.

IL3.13

Small gaps between primes and primes in arithmetic progressions to large moduli

Yitang Zhang
University of New Hampshire, United States of America
yitangz@cisunix.unh.edu

2010 *Mathematics Subject Classification.* 11N05, 11N13

Keywords. Gaps between primes, primes in arithmetic progressions, Bombieri-Vinogradov theorem, Kloostermann sums

Let p_n denote the n-th prime. We describe the proof of the recent result

$$\liminf_{n\to\infty}(p_{n+1}-p_n)<\infty,$$

which is closely related to the distribution of primes in arithmetic progressions to large moduli. A major ingredient of the argument is a stronger version of the Bombieri-Vinogradov theorem which is applicable when the moduli are free from large prime factors.

IL3.12

Linear equations in primes and dynamics of nilmanifolds

Tamar Ziegler
Hebrew University and Technion, Israel
tamarz@math.huji.ac.il

2010 *Mathematics Subject Classification.* 11B30, 37A30, 11B25, 37A45

Keywords. Multiple recurrence, arithmetic progressions, Szemerédi's Theorem, Gowers norms, Hardy-Littlewood conjectures

We survey some of the ideas behind the recent developments in additive number theory, combinatorics and ergodic theory leading to the proof of Hardy-Littlewood type estimates for the number of prime solutions to systems of linear equations of finite complexity.

4. Algebraic Geometry and Complex Geometry

IL4.1

On the virtual fundamental class

Kai Behrend
The University of British Columbia, Canada
behrend@math.ubc.ca

2010 *Mathematics Subject Classification.* 14N35, 14D20
Keywords. Virtual fundamental class, Symmetric obstruction theory, Motivic invariants, Derived geometry

We make a few general remarks about derived schemes, and explain the formalism of the virtual fundamental class. We put particular emphasis on the case of symmetric obstruction theories, and explain why the associated intersection numbers and enumerative invariants (such as those of Donaldson-Thomas) exhibit motivic behaviour. Motivated by this, we raise the question of categorification, and explain why this leads into derived symplectic geometry.

IL4.3

Quasimap theory

Ionuţ Ciocan-Fontanine[1,a] and Bumsig Kim[2,b*]
[1]University of Minnesota, United States of America
[2]Korea Institute for Advanced Study, Republic of Korea
[a]ciocan@math.umn.edu
[b]bumsig@kias.re.kr

2010 *Mathematics Subject Classification.* 14D20, 14D23, 14N35
Keywords. GIT quotients, Quasimaps, Gromov-Witten theory, Mirror symmetry, Gauged linear σ-models

We provide a short introduction to the theory of ε-stable quasimaps and its applications via wall-crossing to Gromov-Witten theory of GIT targets.

IL4.2

Local mirror symmetry in the tropics

Mark Gross[1,a] and Bernd Siebert[2,b*]
[1]University of California at San Diego, United States of America
[2]Universität Hamburg, Germany
[a]mgross@math.ucsd.edu
[b]bernd.siebert@math.uni-hamburg.de

2010 *Mathematics Subject Classification.* 14J33, 14J32, 14M25
Keywords. Mirror symmetry, Toric Calabi-Yau varieties, Tropical geometry

We discuss how the Gross-Siebert reconstruction theorem applies to the local mirror symmetry of Chiang, Klemm, Yau and Zaslow. The reconstruction theorem associates to certain combinatorial data a degeneration of (log) Calabi-Yau varieties. While in this case most of

the subtleties of the construction are absent, an important normalization condition already introduces rich geometry. This condition guarantees the parameters of the construction are canonical coordinates in the sense of mirror symmetry. The normalization condition is also related to a count of holomorphic disks and cylinders, as conjectured in our work and partially proved in various works of Chan, Cho, Lau, Leung and Tseng. We sketch a possible alternative proof of these counts via logarithmic Gromov-Witten theory.

There is also a surprisingly simple interpretation via rooted trees marked by monomials, which points to an underlying rich algebraic structure both in the relevant period integrals and the counting of holomorphic disks.

IL4.4

Semiorthogonal decompositions in algebraic geometry

Alexander Kuznetsov
Steklov Mathematical Institute, Russian Academy of Sciences, Russian Federation
akuznet@mi.ras.ru

2010 *Mathematics Subject Classification.* 18E30, 14F05

Keywords. Semiorthogonal decompositions, exceptional collections, Lefschetz decompositions, homological projective duality, categorical resolutions of singularities, Fano varieties

In this review we discuss what is known about semiorthogonal decompositions of derived categories of algebraic varieties. We review existing constructions, especially the homological projective duality approach, and discuss some related issues such as categorical resolutions of singularities.

IL4.5

K3 surfaces in positive characteristic

Davesh Maulik
Columbia University, United States of America
dmaulik@math.columbia.edu

2010 *Mathematics Subject Classification.* 14J28, 11G25

Keywords. K3 surfaces, Tate conjecture, Moduli spaces

We describe recent progress in the study of K3 surfaces in characteristic p, as well as some geometric applications and open questions.

IL4.6

The dimension of jet schemes of singular varieties

Mircea Mustaţă
University of Michigan, United States of America
mmustata@umich.edu

2010 *Mathematics Subject Classification.* 14E18, 14B05

Keywords. Jet scheme, Log canonical threshold, Minimal log discrepancy

Given a scheme X over k, a generalized jet scheme parametrizes maps $SpecA \to X$, where A is a finite-dimensional, local algebra over k. We give an overview of known results concerning the dimensions of these schemes for $A = k[t]/(t^m)$, when they are related to invariants of singularities in birational geometry. We end with a discussion of more general jet schemes.

IL4.7

Some aspects of explicit birational geometry inspired by complex dynamics

Keiji Oguiso
Graduate School of Science, Osaka University, Japan
oguiso@math.sci.osaka-u.ac.jp

2010 *Mathematics Subject Classification.* 14E07, 14E09, 14D06, 37A35, 37F05

Keywords. Automorphisms, topological entropy, dynamical degrees, rational manifolds, Calabi-Yau manifolds, hyperkaehler manifolds

Our aim is to illustrate how one can effectively apply the basic ideas and notions of topological entropy and dynamical degrees, together with recent progress of minimal model theory in higher dimension, for an explicit study of birational or biregular selfmaps of projective or compact Kaehler manifolds, through concrete examples.

IL4.9

Derived category of coherent sheaves and counting invariants

Yukinobu Toda
Kavli-IPMU, Japan
yukinobu.toda@ipmu.jp

2010 *Mathematics Subject Classification.* 14N35, 18E30

Keywords. Donaldson-Thomas invariants, Bridgeland stability conditions

We survey recent developments on Donaldson-Thomas theory, Bridgeland stability conditions and wall-crossing formula. We emphasize the importance of the counting theory of Bridgeland semistable objects in the derived category of coherent sheaves to find a hidden property of the generating series of Donaldson-Thomas invariants.

IL4.8

Derived algebraic geometry and deformation quantization

Bertrand Toën
CNRS Université de Montpellier 2, France
bertrand.toen@um2.fr

2010 *Mathematics Subject Classification.* 14F05, 18G55

Keywords. Derived Algebraic Geometry, Deformation Quantization

This is a report on recent progress concerning the interactions between derived algebraic geometry and deformation quantization. We present the notion of derived algebraic stacks, of

shifted symplectic and Poisson structures, as well as the construction of deformation quantization of shifted Poisson structures. As an application we propose a general construction of the quantization of the moduli space of G-bundles on an oriented space of arbitrary dimension.

IL4.10

Teichmüller spaces, ergodic theory and global Torelli theorem

Mikhail Verbitsky
National Research University HSE, Moscow, Russian Federation
verbit@verbit.ru

2010 *Mathematics Subject Classification.* 32G13, 53C26

Keywords. Torelli theorem, hyperkahler manifold, moduli space, mapping class group, Teichmüller space

A Teichmüller space T is a quotient of the space of all complex structures on a given manifold M by the connected components of the group of diffeomorphisms. The mapping class group G of M is the group of connected components of the diffeomorphism group. The moduli problems can be understood as statements about the G-action on T. I will describe the mapping class group and the Teichmüller space for a hyperkahler manifold. It turns out that this action is ergodic. We use the ergodicity to show that a hyperkahler manifold is never Kobayashi hyperbolic.

5. Geometry

IL5.4

Family Floer cohomology and mirror symmetry

Mohammed Abouzaid
Columbia University, United States of America
abouzaid@math.columbia.edu

2010 *Mathematics Subject Classification.* 53D40, 14J33

Keywords. Lagrangian Floer cohomology, Homological Mirror symmetry

Ideas of Fukaya and Kontsevich-Soibelman suggest that one can use Strominger-Yau-Zaslow's geometric approach to mirror symmetry as a torus duality to construct the mirror of a symplectic manifold equipped with a Lagrangian torus fibration as a moduli space of simple objects of the Fukaya category supported on the fibres. In the absence of singular fibres, the construction of the mirror is explained in this framework, and, given a Lagrangian submanifold, a (twisted) coherent sheaf on the mirror is constructed.

IL5.2

Hyperbolic orbifolds of small volume

Mikhail Belolipetsky
IMPA, Brazil
mbel@impa.br

2010 *Mathematics Subject Classification.* 22E40, 11E57, 20G30, 51M25

Keywords. Volume, Euler characteristic, hyperbolic manifold, hyperbolic orbifold, arithmetic group

Volume is a natural measure of complexity of a Riemannian manifold. In this survey, we discuss the results and conjectures concerning n-dimensional hyperbolic manifolds and orbifolds of small volume.

IL5.3

Einstein 4-manifolds and singularities

Olivier Biquard
Ecole Normale Superieure, France
olivier.biquard@ens.fr

2010 *Mathematics Subject Classification.* 53C25, 53A30

Keywords. Einstein metric, conformal metric, gravitational instantons, AdS/CFT correspondance

We report on recent progress on the desingularization of real Einstein 4-manifolds. A new type of obstruction is introduced, with applications to the compactification of the moduli space of Einstein metrics, and to the correspondence between conformal metrics in dimension d and asymptotically hyperbolic Einstein metrics in dimension $d + 1$.

IL5.1

Non-negatively curved manifolds and Tits geometry

Fuquan Fang
Capital Normal University, China
fuquan_fang@yahoo.com

2010 *Mathematics Subject Classification.* 53C24, 53C35

Keywords. Curvature, polar action, chamber system, Tits building, Bruhat-Tits building

We explain a surprising passage from non-negatively curved manifolds with polar actions to Tits geometries, which is the basic tool for the rigidity theorem for positively curved polar manifolds established in [FGT], as well as for works in progress on further rigidity theorems for non-negatively curved hyperpolar manifolds. The latter possibly leads to a new characterization of riemannian symmetric spaces.

IL5.5

Loop products, Poincare duality, index growth and dynamics

Nancy Hingston
The College of New Jersey, United States of America
hingston@tcnj.edu

2010 *Mathematics Subject Classification.* 58E10, 55P50

Keywords. Closed geodesics, String topology, Morse theory

A metric on a compact manifold M gives rise to a length function on the free loop space ΛM whose critical points are the closed geodesics on M in the given metric. Morse theory gives a link between Hamiltonian dynamics and the topology of loop spaces, between iteration of closed geodesics and the algebraic structure given by the Chas-Sullivan product on the homology of ΛM. Poincaré Duality reveals the existence of a related product on the cohomology of ΛM.

A number of known results on the existence of closed geodesics are naturally expressed in terms of nilpotence of products. We use products to prove a resonance result for the loop homology of spheres. There are interesting consequences for the length spectrum. We discuss briefly related results in Floer and contact theory.

Mark Goresky and Hans-Bert Rademacher are collaborators.

IL5.7

The surface subgroup and the Ehrenepreis conjectures

Jeremy Kahn[1] and Vladimir Markovic[2,a*]
[1]CUNY, Graduate Center, United States of America
[2]California Institute of Technology, United States of America
[a]markovic@caltech.edu

2010 *Mathematics Subject Classification.* 51

Keywords. 3-manifolds, Hyperbolic Geometry, Dynamics of Geodemtric Flows

We discuss the Surface Subgroup Theorem and the proof of the Ehrenpreis Conjecture

IL5.6

The Geometry of Ricci Curvature

Aaron Naber
Northwestern University, United States of America
anaber@mit.edu

2010 *Mathematics Subject Classification.* 53

Keywords. Ricci curvature, regularity, path space

The talk outlines recent advances in Ricci curvature. Particular focus will be spent on the connections between the Ricci curvature of a manifold and the analysis on the infinite dimensional path space of the manifold. We will see that bounds on the Ricci curvature control the analysis on path space $P(M)$ in a manner very analogous to how lower bounds on the Ricci curvature controls the analysis on M. If time permits more recent advances on the regularity of spaces with bounded Ricci curvature will be discussed, and conjectures about future results will be presented.

IL5.10

New applications of Min-max Theory

André Neves
Imperial College London, United Kingdom
aneves@imperial.ac.uk

2010 *Mathematics Subject Classification.* 53C42, 49Q05

Keywords. Minimal surfaces, Willmore energy, conformal geometry, Min-max Theory

I will talk about my recent work with Fernando Marques where we used Almgren–Pitts Min-max Theory to settle some open questions in Geometry: The Willmore conjecture, the Freedman–He–Wang conjecture for links (jointly with Ian Agol), and the existence of infinitely many minimal hypersurfaces in manifolds of positive Ricci curvature.

IL5.8

When symplectic topology meets Banach space geometry

Yaron Ostrover
Tel Aviv University, Israel
ostrover@post.tau.ac.il

2010 *Mathematics Subject Classification.* 53D35, 52A23, 52A40, 37D50, 57S05

Keywords. Symplectic capacities, Viterbo's volume-capacity conjecture, Mahler's conjecture, Hamiltonian diffeomorphisms, Hofer's metric

In this paper we survey some recent works that take the first steps toward establishing bilateral connections between symplectic geometry and several other fields, namely, asymptotic geometric analysis, classical convex geometry, and the theory of normed spaces.

IL5.9

On the future stability of cosmological solutions to Einstein's equations with accelerated expansion

Hans Ringström
KTH Royal Institute of Technology, Sweden
hansr@kth.se

2010 *Mathematics Subject Classification.* 83C05, 35Q76

Keywords. General relativity, Hyperbolic partial differential equations, Stability, Cosmology, Vlasov matter

The solutions of Einstein's equations used by physicists to model the universe have a high degree of symmetry. In order to verify that they are reasonable models, it is therefore necessary to demonstrate that they are future stable under small perturbations of the corresponding initial data. The purpose of this contribution is to describe mathematical results that have been obtained on this topic. A question which turns out to be related concerns the topology of the universe: what limitations do the observations impose? Using methods similar to ones arising in the proof of future stability, it is possible to construct solutions with arbitrary closed spatial topology. The existence of these solutions indicate that the observations might not impose any limitations at all.

IL5.12

Solitons in geometric evolution equations

Natasa Sesum
Rutgers University, United States of America
natasa.sesum@gmail.com

2010 *Mathematics Subject Classification.* 53C44

Keywords. Ricci flow, Yamabe flow, solitons

We will discuss geometric properties and classification of special solutions to geometric evolution equations called solitons. Our focus will be on the Ricci flow and the Yamabe flow solitons. These are very special solutions to considered geometric evolution equations that move by diffeomorphisms and homotheties. Solitons are very important solutions to our equations because very often they arise as singularity models. Therefore classifying the solitons helps us understand and classify encountered singularities in geometric flows.

IL5.11

Extremal Kähler metrics

Gábor Székelyhidi
University of Notre Dame, United States of America
gszekely@nd.edu

2010 *Mathematics Subject Classification.* 53C55, 14D20, 58E11

Keywords. Extremal Kahler metrics, K-stability, Kahler-Einstein metrics

This paper is a survey of some recent progress on the study of Calabi's extremal Kähler metrics. We first discuss the Yau-Tian-Donaldson conjecture relating the existence of extremal metrics to an algebro-geometric stability notion and we give some example settings where this conjecture has been established. We then turn to the question of what one expects when no extremal metric exists.

IL5.15

Ricci flows with unbounded curvature

Peter M. Topping
University of Warwick, United Kingdom
p.m.topping@warwick.ac.uk

2010 *Mathematics Subject Classification.* 53C44, 35K55, 58J35

Keywords. Ricci flow, Unbounded curvature, Well-posedness, Logarithmic fast diffusion equation, Geometrization

Until recently, Ricci flow was viewed almost exclusively as a way of deforming Riemannian metrics of bounded curvature. Unfortunately, the bounded curvature hypothesis is unnatural for many applications, but is hard to drop because so many new phenomena can occur in the general case. This article surveys some of the theory from the past few years that has sought to rectify the situation in different ways.

IL5.14

Isoperimetric inequalities and asymptotic geometry

Stefan Wenger
University of Fribourg, Switzerland
stefan.wenger@unifr.ch

2010 *Mathematics Subject Classification.* 53C23, 49Q15, 20F65

Keywords. Isoperimetric inequalities, Dehn functions, Gromov hyperbolicity, non-positive curvature, nilpotent groups, Carnot groups, asymptotic cones, currents in metric spaces

The m-th isoperimetric or filling volume function of a Riemannian manifold or a more general metric space X measures how difficult it is to fill an m-dimensional boundary in X of a given volume with an $(m+1)$-dimensional surface in X. The asymptotic growth of the m-th isoperimetric function provides a large scale invariant of the underlying space. Isoperimetric functions have been the subject of intense research in past years in large scale geometry and especially geometric group theory, where they appear as Dehn functions of a group. In this paper and the accompanying talk, I survey relationships between the asymptotic growth of isoperimetric functions and the large scale geometry of the underlying space and, in particular, fine properties of its asymptotic cones. I will furthermore describe recently developed tools from geometric measure theory in metric spaces and explain how these can be used to study the asymptotic growth of the isoperimetric functions.

IL5.13

The cubical route to understanding groups

Daniel Wise
McGill University, Canada
wise@math.mcgill.ca

2010 *Mathematics Subject Classification.* 20F67, 57M99

Keywords. CAT(0) cube complexes, right-angled Artin groups, 3-manifolds

We survey the methodology and key results used to understand certain groups from a cubical viewpoint, and describe the ideas linking 3-manifolds, cube complexes, and right-angled Artin groups. We close with a collection of problems focused on groups acting on CAT(0) cube complexes.

6. Topology

IL6.1

A guide to (étale) motivic sheaves

Joseph Ayoub
University of Zürich, Switzerland
joseph.ayoub@math.uzh.ch

2010 *Mathematics Subject Classification.* 14C25, 14F05, 14F20, 14F42, 18F20

Keywords. Motives, motivic sheaves, motivic cohomology, Grothendieck's six operations, conservation conjecture, motivic t-structures

We recall the construction, following the method of Morel and Voevodsky, of the triangulated category of étale motivic sheaves over a base scheme. We go through the formalism of Grothendieck's six operations for these categories. We mention the relative rigidity theorem. We discuss some of the tools developed by Voevodsky to analyze motives over a base field. Finally, we discuss some long-standing conjectures.

IL6.2

Quasi-morphisms and quasi-states in symplectic topology

Michael Entov
Technion - Israel Institute of Technology, Israel
entov@math.technion.ac.il

2010 *Mathematics Subject Classification.* 53D35, 53D40, 53D45, 17B99, 20F69

Keywords. Symplectic manifold, Hamiltonian symplectomorphism, quantum homology, quasi-morphism, quasi-state

We discuss certain “almost homomorphisms” and “almost linear” functionals that have appeared in symplectic topology and their applications concerning Hamiltonian dynamics, functional-theoretic properties of Poisson brackets and algebraic and metric properties of symplectomorphism groups.

IL6.3

Representation Stability

Benson Farb
University of Chicago, United States of America
bensonfarb@gmail.com

2010 *Mathematics Subject Classification.* 11T06, 14F20, 55N99

Keywords. Configuration space, cohomology, symmetric group, character, representation

Representation stability is a phenomenon whereby the structure of certain sequences X_n of spaces can be seen to stabilize when viewed through the lens of representation theory. In this paper I describe this phenomenon and sketch a framework, the theory of FI-modules, that explains the mechanism behind it.

IL6.4

Moduli spaces of manifolds

Søren Galatius
Stanford University, United States of America
galatius@stanford.edu

2010 *Mathematics Subject Classification.* 57R90, 57R56, 55P47, 57R15
Keywords. Manifolds, Moduli spaces, Diffeomorphism groups, Surgery theory, Infinite loop spaces

This article surveys some recent advances in the topology of moduli spaces, with an emphasis on moduli spaces of manifolds.

IL6.5

On the non-existence of elements of Kervaire invariant one

Michael A. Hill[1,a*] , Michael J. Hopkins[2] , and Douglas C. Ravenel[3]
[1]University of Virginia, United States of America
[2]Harvard University, United States of America
[3]University of Rochester, United States of America
[a]mikehill@virginia.edu

2010 *Mathematics Subject Classification.* 55Q45, 57R60
Keywords. Kervaire invariant, algebraic topology, equivariant homotopy, bordism, slice filtration

We sketch a proof of our solution to the Kervaire invariant one problem, showing that there are Kervaire invariant one manifolds only in dimensions 2, 6, 14, 30, 62, and possibly 126. This resolves a long-standing problem in algebraic and differential topology.

IL6.6

Heegaard splittings of 3-manifolds

Tao Li
Boston College, United States of America
taoli@bc.edu

2010 *Mathematics Subject Classification.* 57N10, 57M50, 57M25
Keywords. Heegaard splitting, 3-manifold

Heegaard splitting is one of the most basic and useful topological structures of 3-manifolds. In the past few years, much progress has been made on Heegaard splittings and several long-standing questions have been answered. We will review some recent progress in studying Heegaard splittings and discuss related open problems.

IL6.7

Isogenies, power operations, and homotopy theory

Charles Rezk
University of Illinois at Urbana-Champaign, United States of America
rezk@math.uiuc.edu

2010 *Mathematics Subject Classification.* 55S25

Keywords. Homotopy theory, formal groups, power operations

The modern understanding of the homotopy theory of spaces and spectra is organized by the chromatic philosophy, which relates phenomena in homotopy theory with the moduli of one-dimensional formal groups. In this paper, we describe how certain phenomena $K(n)$-local homotopy can be computed from knowledge of isogenies of deformations of formal groups of height n.

IL6.8

Algebraic K-theory of strict ring spectra

John Rognes
University of Oslo, Norway
rognes@math.uio.no

2010 *Mathematics Subject Classification.* 19D10, 55P43, 19F27, 57R50

Keywords. Automorphisms of manifolds, brave new rings, logarithmic ring spectrum, motivic truncation, replete bar construction

We view strict ring spectra as generalized rings. The study of their algebraic K-theory is motivated by its applications to the automorphism groups of compact manifolds. Partial calculations of algebraic K-theory for the sphere spectrum are available at regular primes, but we seek more conceptual answers in terms of localization and descent properties. Calculations for ring spectra related to topological K-theory suggest the existence of a motivic cohomology theory for strictly commutative ring spectra, and we present evidence for arithmetic duality in this theory. To tie motivic cohomology to Galois cohomology we wish to spectrally realize ramified extensions, which is only possible after mild forms of localization. One such mild localization is provided by the theory of logarithmic ring spectra, and we outline recent developments in this area.

IL6.9

The topology of scalar curvature

Thomas Schick
Universität Göttingen, Germany
schick@uni-math.gwdg.de

2010 *Mathematics Subject Classification.* 53C21, 53C27, 58D17, 19K56, 46L80

Keywords. Positive scalar curvature, higher index theory, coarse geometry, large scale index theory, C^*-index theory

Given a smooth closed manifold M we study the space of Riemannian metrics of positive scalar curvature on M. A long-standing question is: when is this space non-empty (i.e. when does M admit a metric of positive scalar curvature)? More generally: what is the topology of this space? For example, what are its homotopy groups?

Higher index theory of the Dirac operator is the basic tool to address these questions. This has seen tremendous development in recent years, and in this survey we will discuss some of the most pertinent examples.

In particular, we will show how advancements of large scale index theory (also called coarse index theory) give rise to new types of obstructions, and provide the tools for a systematic study of the existence and classification problem via the K-theory of C^*-algebras. This is part of a program “mapping the topology of positive scalar curvature to analysis”.

In addition, we will show how advanced surgery theory and smoothing theory can be used to construct the first elements of infinite order in the k-th homotopy groups of the space of metrics of positive scalar curvature for arbitrarily large k. Moreover, these examples are the first ones which remain non-trivial in the moduli space.

IL6.10

Gauge theory and mirror symmetry

Constantin Teleman
University of California Berkeley, United States of America
teleman@berkeley.edu

2010 *Mathematics Subject Classification.* 57R56, 55N91, 18D05, 81T13

Keywords. Gauge theory, holomorphic symplectic space, Toda system

Outlined here is a description of equivariance in the world of 2-dimensional extended topological quantum field theories, under a topological action of compact Lie groups. In physics language, I am gauging the theories, coupling them to a principal bundle on the surface worldsheet. I describe the data needed to gauge the theory, as well as the computation of the gauged theory, the result of integrating over all bundles. The relevant theories are ‘A-models’, such as arise from the Gromov-Witten theory of a symplectic manifold with Hamiltonian group action, and the mathematical description starts with a group action on the generating category (the Fukaya category, in this example) which is factored through the topology of the group. Their mirror description involves holomorphic symplectic manifolds and Lagrangians related to the Langlands dual group. An application recovers the complex mirrors of flag varieties proposed by Rietsch.

7. Lie Theory and Generalizations

IL7.2

$\widehat{\mathcal{D}}$-modules on rigid analytic spaces

Konstantin Ardakov
University of Oxford, United Kingdom
ardakov@maths.ox.ac.uk

2010 *Mathematics Subject Classification.* 14G22, 16S38, 22E50, 32C38

Keywords. $\mathcal{D}$-modules, rigid analytic geometry, Beilinson-Bernstein localisation, locally analytic representations, p-adic Lie groups

We give an overview of the theory of $\widehat{\mathcal{D}}$-modules on rigid analytic spaces and its applications to admissible locally analytic representations of p-adic Lie groups.

IL7.5

Boundaries, rigidity of representations, and Lyapunov exponents

Uri Bader[1,a] and Alex Furman[2*]
[1]University of Illinois at Chicago, United States of America
[2]Technion, Israel
[a]furman@math.uic.edu

2010 *Mathematics Subject Classification.* 37A, 22E

Keywords. Boundary theory, isometric ergodicity, characteristic maps, superrigidity, Lyapunov exponents

In this paper we discuss some connections between measurable dynamics and rigidity aspects of group representations and group actions. A new ergodic feature of familiar group boundaries is introduced, and is used to obtain rigidity results for group representations and to prove simplicity of Lyapunov exponents for some dynamical systems.

IL7.1

Recurrence on the space of lattices

Yves Benoist
CNRS - Universite Paris-Sud, France
yves.benoist@math.u-psud.fr

2010 *Mathematics Subject Classification.* 22E40, 37C85, 60J05

Keywords. Lie groups, discrete subgroups, homogeneous dynamics, Markov chains

This is an introduction to recurrence properties on finite volume homogeneous spaces based on examples.

IL7.3

Diophantine geometry and uniform growth of finite and infinite groups

Emmanuel Breuillard
Université Paris-Sud 11, France
emmanuel.breuillard@math.u-psud.fr

2010 *Mathematics Subject Classification.* 22E40, 11G50

Keywords. Diophantine geometry, heights, small points, Lehmer conjecture, exponential growth, amenability, Tits alternative, spectral gaps, approximate groups.

We survey a number of recent results regarding the geometry and spectra of finite and infinite groups. In particular we discuss the uniform Tits alternative for infinite linear groups highlighting the inputs from diophantine geometry and the consequences for finite groups.

IL7.6

Schur-Weyl duality and categorification

Jonathan Brundan
University of Oregon, United States of America
brundan@uoregon.edu

2010 *Mathematics Subject Classification.* 17B10, 16G99, 18D99

Keywords. Schur-Weyl duality, highest weight category, categorification

In some joint work with Kleshchev in 2008, we discovered a higher level analog of Schur-Weyl duality, relating parabolic category $\mathcal{O}$ for the general linear Lie algebra to certain cyclotomic Hecke algebras. Meanwhile Rouquier and others were developing a general axiomatic approach to the study of categorical actions of Lie algebras. In this survey, we recall aspects of these two theories, then explain some related recent developments due to Losev and Webster involving tensor product categorifications.

IL7.7

Modular representation theory of symmetric groups

Alexander Kleshchev
University of Oregon, United States of America
klesh@uoregon.edu

2010 *Mathematics Subject Classification.* 20C30, 20C08, 17B37

Keywords. Symmetric group, modular representation, Hecke algebra, KLR algebra, quantum group

We review some recent advances in modular representation theory of symmetric groups and related Hecke algebras. We discuss connections with Khovanov-Lauda-Rouquier algebras and gradings on the blocks of the group algebras of symmetric groups, which these connections reveal; graded categorification and connections with quantum groups and crystal bases; modular branching rules and the Mullineaux map; graded cellular structure and graded Specht modules; cuspidal systems for affine KLR algebras and imaginary Schur-Weyl duality, which connects representation theory of these algebras to the usual Schur algebras of smaller rank.

IL7.8

Multi-fusion categories of Harish-Chandra bimodules

Victor Ostrik
University of Oregon, United States of America
vostrik@uoregon.edu

2010 *Mathematics Subject Classification.* 17B35, 18D10, 22E47, 14F05
Keywords. Harish-Chandra modules, tensor categories, finite W-algebras

We survey some results on tensor products of irreducible Harish-Chandra bimodules. It turns out that such tensor products are semisimple in suitable Serre quotient categories. We explain how to identify the resulting semisimple tensor categories and describe some applications to representation theory.

IL7.10

On some recent developments in the theory of buildings

Bertrand Rémy
Université Claude Bernard Lyon 1, France
remy@math.univ-lyon1.fr

2010 *Mathematics Subject Classification.* 51E24, 20E42, 20E32, 20F65, 14G22
Keywords. Building, Algebraic group, Bruhat-Tits theory, Kac-Moody theory, Simplicity

Buildings are cell complexes with so remarkable symmetry properties that many groups from important families act on them. We present some examples of results in Lie theory and geometric group theory obtained thanks to these highly transitive actions. The chosen examples are related to classical and less classical (often non-linear) group-theoretic situations.

IL7.9

Some qualitative properties of branching multiplicities

Nicolas Ressayre
Université Claude Bernard Lyon 1, France
ressayre@math.univ-lyon1.fr

2010 *Mathematics Subject Classification.* 14L24, 14N15, 14N35, 22C99
Keywords. Branching multiplicities, additive and multiplicative eigenvalue problems, (quantum) Schubert calculus

Let G be a connected reductive subgroup of a complex connected reductive group $\hat{G}$. We consider the multiplicities $c_{G,\hat{G}}$ as a function from the set of pairs of dominant weights to the set of integers. We recall that this function is piecewise quasipolynomial. Its support is a finitely generated semigroup ; we describe an irredundant list of inequalities determining the cone generated. The relation with the projection of coadjoint orbits for the Lie algebras of the compact forms of G and $\hat{G}$ is also recalled.

We also consider the multiplicities for the fusion products for G. More precisely, we explain how the small quantum cohomology rings of homogeneous spaces G/P allow to

parametrize an irredundant set of inequalities determining the multiplicative eigenvalue problem for the compact form K of G.

IL7.4

Double affine Hecke algebras and Hecke algebras associated with quivers

Michela Varagnolo[1] and Eric Vasserot[2,a*]
[1]Université de Cergy-Pontoise, France
[2]Institut de Mathématiques de Jussieu, France
[a]vasserot@math.jussieu.fr

2010 *Mathematics Subject Classification.* 06B15
Keywords. Hecke algebras

This is a short survey of some geometrical and categorical approaches to the representation theory of several algebras related to Hecke algebras, including cyclotomic Hecke algebras, double affine Hecke algebras and quiver-Hecke algebras.

8. Analysis and its Applications

IL8.1

Random matrices, log-gases and Hölder regularity

László Erdős
Institute of Science and Technology, Austria

2010 *Mathematics Subject Classification.* 15B52, 82B44

Keywords. De Giorgi-Nash-Moser parabolic regularity, Wigner-Dyson-Gaudin-Mehta universality, Dyson Brownian motion

The Wigner-Dyson-Gaudin-Mehta conjecture asserts that the local eigenvalue statistics of large real and complex Hermitian matrices with independent, identically distributed entries are universal in a sense that they depend only on the symmetry class of the matrix and otherwise are independent of the details of the distribution. We present the recent solution to this half-century old conjecture. We explain how stochastic tools, such as the Dyson Brownian motion, and PDE ideas, such as De Giorgi-Nash-Moser regularity theory, were combined in the solution. We also show related results for log-gases that represent a universal model for strongly correlated systems. Finally, in the spirit of Wigner's original vision, we discuss the extensions of these universality results to more realistic physical systems such as random band matrices.

IL8.2

Quantitative stability results for the Brunn-Minkowski inequality

Alessio Figalli
The University of Texas at Austin, United States of America
figalli@math.utexas.edu

2010 *Mathematics Subject Classification.* 49Q20, 35A23

Keywords. Geometric and functional inequalities, quantitative stability, sumsets, Brunn-Minkowski

The Brunn-Minkowski inequality gives a lower bound of the Lebesgue measure of a sumset in terms of the measures of the individual sets. This inequality plays a crucial role in the theory of convex bodies and has many interactions with isoperimetry and functional analysis. Stability of optimizers of this inequality in one dimension is a consequence of classical results in additive combinatorics. In this note we describe how optimal transportation and analytic tools can be used to obtain quantitative stability results in higher dimension.

IL8.3

Q and Q-prime curvature in CR geometry

Kengo Hirachi
The University of Tokyo, Japan
hirachi@ms.u-tokyo.ac.jp

2010 *Mathematics Subject Classification.* 32V05, 32T15

Keywords. CR geometry, Q-curvature, Parabolic geometry, conformal geometry, strictly pseudoconvex domain

The Q-curvature has been playing a central role in conformal geometry since its discovery by T. Branson. It has natural analogy in CR geometry, however, the CR Q-curvature vanishes on the boundary of a strictly pseudoconvex domain in $\mathbb{C}^{n+1}$ with a natural choice of contact form. This fact enables us to define a "secondary" Q-curvature, which we call Q-prime curvature (it was first introduced by J. Case and P. Yang in the case $n = 1$). The integral of the Q-prime curvature, the total Q-prime curvature, is a CR invariant of the boundary. When $n = 1$, it agrees with the Burns-Epstein invariant, which is a Chern-Simons type invariant in CR geometry. For all $n \geq 1$, it has non-trivial variation under the deformation of domains. Combining the variational formula with the deformation complex of CR structures, we show that the total Q-prime curvature takes local maximum at the standard CR sphere in a formal sense. This talk is a report in collaboration with Rod Gover, Yoshihiko Matsumoto, Taiji Marugame and Bent Orsted.

IL8.4

Advances in weighted norm inequalities

Tuomas Hytönen
University of Helsinki, Finland
tuomas.hytonen@helsinki.fi

2010 *Mathematics Subject Classification.* 42B20, 42B25

Keywords. Weighted norm inequality, A_2 theorem, two-weight problem, Hilbert transform, testing condition

The classical theory of weighted norm inequalities provides a characterization of admissible weights such that the Hilbert transform or other singular operators act boundedly from the weighted space $L^p(w)$ to itself. This lecture surveys two lines of recent development: proving sharp quantitative forms of the classical mapping properties (the A_2 theorem), and characterizing the admissible pairs of weights when the operator acts from one $L^2(u)$ space to another $L^p(v)$ (the two-weight problem).

IL8.5

The flecnode polynomial: a central object in incidence geometry

Nets Hawk Katz
California Institute of Technology, United States of America
nets@caltech.edu

2010 *Mathematics Subject Classification.*

Keywords. Flecnode

We describe recent advances in the theory of incidence geometry. We focus on the role of the flecnode polynomial of Cayley and Salmon

IL8.6

Harmonic analysis and the geometry of fractals

Izabella Łaba
The University of British Columbia, Canada
ijlaba@gmail.com

2010 *Mathematics Subject Classification.* 28A78, 42A38, 42B25, 26A24, 11B25
Keywords. Fourier analysis, Hausdorff dimension, restriction estimates, maximal operators

Singular and oscillatory integral estimates such as maximal theorems and restriction estimates for measures on hypersurfaces have long been a central topic in harmonic analysis. We discuss the recent work by the author and her collaborators on the analogues of such results for singular measures supported on fractal sets. The common thread is the use of ideas from additive combinatorics. In particular, the additive-combinatorial notion of "pseudorandomness" for fractals turns out to be an appropriate substitute for the curvature of manifolds.

IL8.14

Mean field equations, hyperelliptic curves and modular forms

Chang-Shou Lin
National Taiwan University, Taiwan
cslin@math.ntu.edu.tw

2010 *Mathematics Subject Classification.* 35A01, 35A02, 35B38, 35B44, 35J08
Keywords. Mean field equation, Green function, Lame equation, Hyperelliptic curve, Modular form

In this paper, we survey some recent joint works with C. L. Chai and C. L. Wang, in which we have developed a theory to connect the mean field equation, Green function and Lame equation. In this theory, we have constructed a family of hyperelliptic curves and a premodular form of degree $0.5n(n+1)$ and proved that the nonlinear elliptic PDE on a flat torus Et has a solution iff t is a zero of this pre-modular form. As a consequence, we show that the Green function of the torus has either three critical points or five critical points. Furthermore, the set of tori in the moduli space such that G has five critical points is simply-connected.

IL8.10

Liouville equations from a variational point of view

Andrea Malchiodi
Scuola Internazionale Superiore di Studi Avanzati, Italy
malchiod@sissa.it

2010 *Mathematics Subject Classification.* 35J20, 35J75, 35Q40, 53A30
Keywords. Liouville equations, variational methods, conformal geometry, singular PDEs

After discussing the role of Liouville equations in both Conformal Geometry and Mathematical Physics, we will explore some of their variational features. In particular we will show the role of the Moser-Trudinger inequality, as well as of some of its improved versions, in characterizing the Euler-Lagrange energy levels of the problems under interest. This descrip-

tion reduces the study of PDEs of Liouville type to topological properties of explicit finite-dimensional objects.

IL8.15

Ramanujan graphs and the solution of the Kadison–Singer problem

Adam W. Marcus[1,a] , Daniel A. Spielman[2,b] , and Nikhil Srivastava[3*]
[1]Yale University and Crisply, Inc., United States of America
[2]Yale University, United States of America
[3]Microsoft Research, India
[a]adam.marcus@yale.edu
[b]spielman@cs.yale.edu

2010 *Mathematics Subject Classification.* 05C50, 46L05, 26C10

Keywords. Interlacing polynomials, Kadison–Singer, Ramanujan graphs, restricted invertibility, mixed characteristic polynomials

We survey the techniques used in our recent resolution of the Kadison–Singer problem and proof of existence of Ramanujan Graphs of every degree: mixed characteristic polynomials and the method of interlacing families of polynomials. To demonstrate the method of interlacing families of polynomials, we give a simple proof of Bourgain and Tzafriri's restricted invertibility principle in the isotropic case.

IL8.12

Carleson measures and elliptic boundary value problems

Jill Pipher
Brown university, United States of America
jill_pipher@brown.edu

2010 *Mathematics Subject Classification.* 42B99, 42B25, 35J25, 42B20

Keywords. Carleson measures, elliptic divergence form operators, boundary value problems

In this article, we highlight the role of Carleson measures in elliptic boundary value problems, and discuss some recent results in this theory. The focus here is on the Dirichlet problem, with measurable data, for second order elliptic operators in divergence form. We illustrate, through selected examples, the various ways Carleson measures arise in characterizing those classes of operators for which Dirichlet problems are solvable with classical non-tangential maximal function estimates.

IL8.9

Roth's theorem: an application of approximate groups

Tom Sanders
University of Oxford, United Kingdom
tom.sanders@maths.ox.ac.uk

2010 *Mathematics Subject Classification.*

Keywords. Approximate groups, Roth's theorem, additive combinatorics, Bourgain systems

We discuss Roth's theorem on arithmetic progressions through the lens of approximate groups.

IL8.8

Semilinear wave equations

Wilhelm Schlag
The University of Chicago, United States of America
schlag@math.uchicago.edu

2010 *Mathematics Subject Classification.* 35L05, 35L52, 37K40, 37K45, 53Z05

Keywords. Semilinear wave equations, asymptotic stability, wellposedness, wave maps

We will review some of the recent work on semilinear wave equations, in particular the wave map equation. We discuss global wellposedness, as well as the construction of special solutions and their stability.

IL8.11

Several applications of the moment method in random matrix theory

Sasha Sodin
Princeton University, United States of America and Tel Aviv University, Israel
asodin@princeton.edu

2010 *Mathematics Subject Classification.* 60B20, 44A60

Keywords. Moment method, Random matrices, Orthogonal polynomials

Several applications of the moment method in random matrix theory, especially, to local eigenvalue statistics at the spectral edges, are surveyed, with emphasis on a modification of the method involving orthogonal polynomials.

IL8.13

Free probability and random matrices

Roland Speicher
Universität des Saarlandes, Germany
speicher@math.uni-sb.de

2010 *Mathematics Subject Classification.* 46L54, 60B20

Keywords. Free probability, random matrices, linearization trick, free cumulants, operator-valued free probability

The concept of freeness was introduced by Voiculescu in the context of operator algebras. Later it was observed that it is also relevant for large random matrices. We will show how the combination of various free probability results with a linearization trick allows to address successfully the problem of determining the asymptotic eigenvalue distribution of general selfadjoint polynomials in independent random matrices.

IL8.7

The h-principle and turbulence

László Székelyhidi Jr.
Universität Leipzig, Germany
szekelyhidi@math.uni-leipzig.de

2010 *Mathematics Subject Classification.* 35D30, 76F05, 34A60, 53B20

Keywords. Euler equations, anomalous dissipation, h-principle, Onsager's conjecture

It is well known since the pioneering work of Scheffer and Shnirelman that weak solutions of the incompressible Euler equations exhibit a wild behaviour, which is very different from that of classical solutions. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of Lars Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to the K41 theory of turbulence.

In recent joint work with Camillo De Lellis we established a connection between the theory of weak solutions of the Euler equations and the Nash-Kuiper theorem on rough isometric immersions. Through this connection we interpret the wild behaviour of weak solutions of Euler as an instance of Gromov's h-principle. In this lecture we explain this connection and outline recent progress towards Onsager's conjecture.

9. Dynamical Systems and Ordinary Differential Equations

IL9.1

Linear response, or else

Viviane Baladi
CNRS, DMA-Ecole Normale Supérieure, Paris, France
viviane.baladi@ens.fr

2010 *Mathematics Subject Classification.* 37C40, 37D25, 37C30, 37E05

Keywords. Linear response, Transfer operator, SRB measure, Unimodal maps, Hyperbolic dynamical systems

Consider a smooth one-parameter family $t \mapsto f_t$ of dynamical systems f_t, with $|t| < \epsilon$. Assume that for all t (or for many t close to $t = 0$) the map f_t admits a unique physical invariant probability measure μ_t. We say that linear response holds if $t \mapsto \mu_t$ is differentiable at $t = 0$ (possibly in the sense of Whitney), and if its derivative can be expressed as a function of f_0, μ_0, and $\partial_t f_t|_{t=0}$. The goal of this note is to present to a general mathematical audience recent results and open problems in the theory of linear response for chaotic dynamical systems, possibly with bifurcations.

IL9.2

Metric stability of the planetary N-body problem

Luigi Chierchia[1,a*] and Gabriella Pinzari[2]
[1]Universitá degli Studi Roma Tre, Italy
[2]Universitá Federico II, Napoli, Italy
[a]luigi@mat.uniroma3.it

2010 *Mathematics Subject Classification.* 70H08, 70K43, 70F10, 70H12, 70K45

Keywords. Planetary system, N-body problem, Metric stability, Quasi-periodic motions, Symplectic invariants

The "solution" of the N-body problem (NBP) has challenged astronomers and mathematicians for centuries. In particular, the "metric stability" (i.e., stability in a suitable measure theoretical sense) of the planetary NBP is a formidable achievement in this subject completing an intricate path paved by mathematical milestones (by Newton, Weierstrass, Lindstedt, Poincarè, Birkhoff, Siegel, Kolmogorov, Moser, Arnold, Herman,…). In 1963 V.I. Arnold gave the following formulation of the metric stabiliy of the planetary problem:
If the masses of n planets are sufficiently small in comparison with the mass of the central body, the motion is conditionally periodic for the majority of initial conditions for which the eccentricities and inclinations of the Kepler ellipses are small.
Arnold gave a proof of this statement in a particular case (2 planets in a plane) and outlined a strategy (turned out to be controversial) for the general case. Only in 2004 J. Féjoz, completing work by M.R. Herman, published the first proof of Arnold's statement following a different approach using a "first order KAM theory" (developed by Rüssmann, Herman et al., and based on weaker non-degeneracy conditions) and removing certain secular degeneracies by the aid of an auxiliary fictitious system. Arnold's more direct and powerful strategy – including proof of torsion, Birkhoff normal forms, explicit measure estimates – has been completed in 2011 by the authors introducing new symplectic coordinates, which allow, after

a proper symplectic reduction of the phase space, a direct check of classical non–degeneracy conditions.

IL9.6

Dynamics of C^1-diffeomorphisms: global description and prospects for classification

Sylvain Crovisier
University Paris-Sud 11, France
Sylvain.Crovisier@math.u-psud.fr

2010 *Mathematics Subject Classification.* 37C20, 37C50, 37D25, 37D30, 37C29

Keywords. Differentiable dynamical systems, closing lemma, homoclinic bifurcation, partial hyperbolicity, generic dynamics

We are interested in finding a dense part of the space of C^1-diffeomorphisms which decomposes into open subsets corresponding to different dynamical behaviors: we discuss results and questions in this direction.

In particular we present recent results towards a conjecture by J. Palis: any system can be approximated either by one which is hyperbolic (and whose dynamics is well un- derstood) or by one which exhibits a homoclinic bifurcation (a simple local configuration involving one or two periodic orbits).

IL9.5

Weak KAM Theory: the connection between Aubry-Mather theory and viscosity solutions of the Hamilton-Jacobi equation

Albert Fathi
ENS de Lyon, France
albert.fathi@ens-lyon.fr

2010 *Mathematics Subject Classification.* 37J50, 35F21, 70H20

Keywords. Lagrangian, Hamiltonian, Hamilton-Jacobi, Aubry-Mather, weak KAM

The goal of this lecture is to explain to the general mathematical audience the connection that was discovered in the last 20 or so years between the Aubry-Mather theory of Lagrangian systems, due independently to Aubry and Mather in low dimension, and to Mather in higher dimension, and the theory of viscosity solutions of the Hamilton-Jacobi equation, due to Crandall and Lions, and more precisely the existence of global viscosity solutions due to Lions, Papanicolaou, and Varhadan.

IL9.3

Resonances for geodesic flows on negatively curved manifolds

Frédéric Faure[1,a] and Masato Tsujii[2,b*]
[1]Institut Fourier, France
[2]Kyushu university, Japan
[a]frederic.faure@ujf-grenoble.fr
[b]tsujii@math.kyushu-u.ac.jp

2010 *Mathematics Subject Classification.* 37C30, 37D40, 81Q50

Keywords. Geodesic flow, Transfer operator, Dynamical zeta function, Ruelle-Pollicott resonance, Anosov flow

We report some recent progress in the study of geodesic flows on negatively curved manifolds (or more generally contact Anosov flows). We consider one-parameter groups of transfer operators associated to the flows and investigate the spectra of their generators. The main ingredients are the recent results about a band structure of the discete spectrum, which are obtained in the author's joint works with F. Faure (Fourier Institure, Grenoble, France).

IL9.4

The low-density limit of the Lorentz gas: periodic, aperiodic and random

Jens Marklof
University of Bristol, United Kingdom
j.marklof@bristol.ac.uk

2010 *Mathematics Subject Classification.* 82C40, 35Q20, 37A17, 37D50, 60G55

Keywords. Boltzmann equation, Boltzmann-Grad limit, homogeneous flow, Lorentz gas, quasicrystal

The Lorentz gas is one of the simplest, most widely used models to study the transport properties of rarified gases in matter. It describes the dynamics of a cloud of non-interacting point particles in an infinite array of fixed spherical scatterers. More than one hundred years after its conception, it is still a major challenge to understand the nature of the kinetic transport equation that governs the macroscopic particle dynamics in the limit of low scatterer density (the Boltzmann-Grad limit). Lorentz suggested that this equation should be the linear Boltzmann equation. This was confirmed in three celebrated papers by Gallavotti, Spohn, and Boldrighini, Bunimovich and Sinai, under the assumption that the distribution of scatterers is sufficiently disordered. In the case of strongly correlated scatterer configurations (such as crystals or quasicrystals), we now understand why the linear Boltzmann equation fails and what to substitute it with. A particularly striking feature of the periodic Lorentz gas is a heavy tail for the distribution of free path lengths, with a diverging second moment, and superdiffusive transport in the limit of large times.

IL9.7

Fractal geometry and dynamical bifurcations

Carlos Gustavo T. de A. Moreira
IMPA, Brazil
gugu@impa.br

2010 *Mathematics Subject Classification.* 37C29, 28A80, 28A78, 11J06

Keywords. Fractal geometry, Homoclinic bifurcations, Diophantine approximations

In this survey we will describe results that relate qualitative properties of dynamical systems (and bifurcations of dynamical systems) to geometrical properties of invariant sets of these

systems - fractal dimensions of hyperbolic invariant sets have a key role in such results. We conclude with a discussion of some results on geometrical properties of the classical Markov and Lagrange spectra of diophantine approximations and of dynamical variations of them, which can be proved using techniques of fractal geometry developed primarily in the context of dynamical bifurcations mentioned above.

IL9.8

Zeta functions for Anosov flows

Mark Pollicott
Warwick University, United Kingdom
masdbl@warwick.ac.uk

2010 *Mathematics Subject Classification.* 37C30
Keywords. Dynamical Zeta Functions, Thermodynamical Formalism

Dynamical zeta functions, by analogy with their more famous counterparts in number theory, are a useful tool to study certain types of dynamical systems. An important application is to the geodesic flow on a negatively curved surface. For surfaces of constant negative curvature the properties of the Selberg zeta function have been well understood for over half a century. However, understanding the properties of the corresponding zeta function for the more general setting of surfaces of variable negative curvature benefits from this more dynamical viewpoint.

IL9.9

Recent developments in interval dynamics

Weixiao Shen[1,a] and Sebastian van Strien[2,b*]
[1]National University of Singapore, Singapore
[2]Imperial College, United Kingdom
[a]matsw@nus.edu.sg
[b]s.van-strien@imperial.ac.uk

2010 *Mathematics Subject Classification.* 37F10, 30D05
Keywords. One-dimensional dynamics, Dynamics, Ergodic theory

Dynamics in dimension-one has been an extremely active research area over the last decades. In this talk we will describe some of the new developments of the recent years.

10. Partial Differential Equations

IL10.8

From molecular dynamics to kinetic theory and hydrodynamics

Thierry Bodineau[1] , Isabelle Gallagher[2,a*] , and Laure Saint-Raymond[3]
[1]CNRS and Ecole polytechnique, France
[2]Université Paris-Diderot, France
[3]Ecole Normale Superieure, France
[a]gallagher@math.univ-paris-diderot.fr

2010 *Mathematics Subject Classification.* 76P05, 82B21, 82B40

Keywords. Boltzmann equation, BBGKY hierarchy, heat equation ; Brownian motion, branching process, low density limit

In these notes we present the main ingredients of the proof of the convergence of the distribution function of a tagged particle in a background initially at equilibrium, towards the solution to the heat equation. We also show how the process associated with the tagged particle converges in law towards a Brownian motion.

IL10.7

From molecular dynamics to kinetic theory and hydrodynamics

Thierry Bodineau[1,a] , Isabelle Gallagher[2,b] , and Laure Saint-Raymond[3,c*]
[1]CNRS & École Polytechnique, France
[2]Université Paris-Diderot, France
[3]École Normale Supérieure, France
[a]Thierry.Bodineau@polytechnique.edu
[b]gallagher@math.univ-paris-diderot.fr
[c]Laure.Saint-Raymond@ens.fr

2010 *Mathematics Subject Classification.* 76P05, 82B21, 82B40

Keywords. Boltzmann equation, BBGKY hierarchy, heat equation ; Brownian motion, branching process, low density limit

In these notes we present the main ingredients of the proof of the convergence of the distribution function of a tagged particle in a background initially at equilibrium, towards the solution to the heat equation. We also show how the process associated with the tagged particle converges in law towards a Brownian motion.

IL10.2

The mathematical analysis of black holes in general relativity

Mihalis Dafermos
University of Cambridge, United Kingdom
Thierry.Bodineau@polytechnique.edu

2010 *Mathematics Subject Classification.* 83C57, 83C75

Keywords. Einstein equations, general relativity, black holes, cosmic censorship

The mathematical analysis of black holes in general relativity has been the focus of con-

siderable activity in the past decade from the perspective of the theory of partial differential equations. Much of this work is motivated by the problem of understanding the two celebrated cosmic censorship conjectures in a neighbourhood of the Schwarzschild and Kerr solutions. Recent progress on the behaviour of linear waves on black hole exteriors as well as on the full non-linear vacuum dynamics in the black hole interior puts us at the threshold of a complete understanding of the stability–and instability–properties of these solutions. This talk will survey some of these developments.

IL10.1

Ancient solutions to geometric flows

Panagiota Daskalopoulos
Columbia University, United States of America
pdaskalo@math.columbia.edu

2010 *Mathematics Subject Classification.* 53C44, 35K55, 35B08

Keywords. Ancient solutions, Geometric evolution equations, Ricci flow, Yamabe flow

We will discuss ancient and eternal solutions to geometric parabolic equations. These are special solutions that exist for time $-\infty < t \le T$, with $T \le +\infty$. They often appear as blow up limits near a singularity. Their classification often results to the better understanding of the singularities of the flow. We will address the classification of ancient solutions to the Ricci flow on surfaces and the Yamabe flow on S^n and point out future open directions. The results in this article are joint work of the author with the collaborators M. del Pino, R. Hamilton and N. Sesum.

IL10.4

Quantized vortex filaments in complex scalar fields

Robert L. Jerrard
University of Toronto, Canada
rjerrard@math.toronto.edu

2010 *Mathematics Subject Classification.* 35B40, 35B25, 49Q15

Keywords. Semilinear PDEs, singular limit, geometric measure theory, vortex filament, Ginzburg-Landau

We survey a family of problems in which one seeks to prove that, for a complex-valued function solving a semilinear partial differential equation, energy concentrates, in certain scaling limits, around a codimension 2 submanifold solving a geometric problem. The equations in question arise from physical models, and the energy concentration sets are often naturally interpreted as "quantized vortex filaments." One can hope to describe these vortex filaments in a variety of types of PDE, including elliptic (describing an equilibrium of a physical system), parabolic (often describing flow toward an equilibrium) and hyperbolic or dispersive (describing different kinds of oscillations and wave propagation). There are a large number of results about elliptic and parabolic equations, although some significant open problems remain, and less is known about hyperbolic and (especially) dispersive equations.

IL10.11

The resolution of the bounded L^2 curvature conjecture in general relativity

Sergiu Klainerman[1,a] , Igor Rodnianski[1,b] , and Jérémie Szeftel[2,c*]
[1]Princeton University, United States of America
[2]Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie, France
[a]seri@math.princeton.edu
[b]irod@math.princeton.edu & irod@math.mit.edu
[c]jeremie.szeftel@upmc.fr

2010 *Mathematics Subject Classification.* 83C05, 35Q75, 58J45

Keywords. Einstein equations, Cauchy problem, rough solutions, null structure, bilinear estimates

In order to control locally a space-time which satisfies the Einstein equations, what are the minimal assumptions one should make on its curvature tensor? The bounded L^2 curvature conjecture roughly asserts that one should only need L^2 bounds of the curvature tensor on a given space-like hypersurface. This conjecture has its roots in the remarkable developments of the last twenty years centered around the issue of optimal well-posedness for nonlinear wave equations. In this context, a corresponding conjecture for nonlinear wave equations cannot hold, unless the nonlinearity has a very special nonlinear structure. I will present the proof of this conjecture, which sheds light on the specific null structure of the Einstein equations. This is joint work with Sergiu Klainerman and Igor Rodnianski.

IL10.5

Homogenization Theory in Nonlinear Partial Differential Equations

Ki-Ahm Lee
Seoul National University, Republic of Korea
kiahm@snu.ac.kr

2010 *Mathematics Subject Classification.* 35B27, 35K55, 35R35

Keywords. Homogenization, Nonlinear Partial Differential Equations, Viscosity Method, Free Boundary Problems, Perforated Domains

In this talk, we are going to discuss recent works in Homogenization Theory in Nonlinear Partial Differential Equations, which concerns the derivation of a macroscopic homogenized (or effective) equation for the system with oscillating microscopic structures. Mainly we focus on topics related with the homogenization for high oscillation, nonvariational problems, lower dimensional oscillations or equations of nondivergence type.

IL10.6

Academic wages, singularities, phase transitions and pyramid schemes

Robert J. McCann
University of Toronto, Canada
mccann@math.toronto.edu

2010 *Mathematics Subject Classification.* 91B68, 49N15, 35Q91, 91B66, 91B69

Keywords. Multisector matching with heterogeneous types, steady-state competitve equilibrium, education and labor markets, optimal transportation, infinite-dimensional linear program

In this lecture we introduce a mathematical model which couples the education and labor markets, in which steady-steady competitive equilibria turn out to be characterized as the solutions to an infinite-dimensional linear program and its dual. In joint work with Erlinger, Shi, Siow and Wolthoff, we use ideas from optimal transport to analyze this program, and discover the formation of a pyramid-like structure with the potential to produce a phase transition separating singular from non-singular wage gradients.

Wages are determined by supply and demand. In a steady-state economy, individuals will choose a profession, such as worker, manager, or teacher, depending on their skills and market conditions. But these skills are determined in part by the education market. Some individuals participate in the education market twice, eventually marketing as teachers the skills they acquired as students. When the heterogeneity amongst student skills is large, so that it can be modeled as a continuum, this feedback mechanism has the potential to produce larger and larger wages for the few most highly skilled individuals at the top of the market. We analyze this phenomena using the aforementioned model. We show that a competitive equilibrium exists, and it displays a phase transition from bounded to unbounded wage gradients, depending on whether or not the impact of each teacher increases or decreases as we pass through successive generations of their students. We specify criteria under which this equilibrium will be unique, and under which the educational matching will be positive assortative. The latter turns out to depend on convexity of the equilibrium wages as a function of ability, suitably parameterized.

IL10.10

On singularity formation in Hamiltonian evolution equations

Pierre Raphaël
Universite de Nice Sophia-Antipolis, France
praphael@unice.fr

2010 *Mathematics Subject Classification.* 35Q41, 35Q51, 35Q53, 35Q55, 37Q40

Keywords. Hamiltonian evolution equations, non linear Schrödinger equations, singularity formation, solitons

Hamiltonian evolution equations arise in the description of nonlinear phenomenons in various instances from nonlinear optics to astrophysics or fluid mechanics, but the description of most even simplified models still remains a mathematical challenge. Substantial progress have been made since the 1980's for the qualitative description of solutions through the importation and mixing of various ideas from dynamical systems, functional analysis, harmonic analysis and the calculus of variations. I will report in this survey on recent progress on the study of one specific scenario: singularity formation, that is the ability for non linear waves to concentrate their energy while propagating in some nonlinear medium. A new methodology has emerged in the last two decades on canonical models like the non linear Schrödinger or wave equations both for the construction and the classification of singular regimes, with applications also to parabolic models. A special class of solutions plays a distinguished role in the structure of the corresponding blow up bubbles: the solitary wave.

IL10.3

Regularity estimates for parabolic integro-differential equations and applications

Luis Silvestre
University of Chicago, United States of America
luis@math.uchicago.edu

2010 *Mathematics Subject Classification.* 45E99, 35K99
Keywords. Non local equations, regularity

We review some regularity results for integro-differential equations, focusing on Holder estimates for equations with rough kernels and their applications. These equations are a natural generalization of second order elliptic and parabolic partial differential equations. We show how we can obtain a regularization effect from the singular integral structure of the equation. We obtain Holder estimates even in cases where the Harnack inequality does not hold. The regularity estimates have applications to models from probability and fluids among others.

IL10.12

Some recent advances in microlocal analysis

András Vasy
Stanford University, United States of America
andras@math.stanford.edu

2010 *Mathematics Subject Classification.* 35S05, 35A21, 35P25, 35L05, 58J50
Keywords. Pseudodifferential operators, radial points, manifolds with boundary, non-elliptic Fredholm theory, Kerr-de Sitter space

In this talk we describe some recent developments in microlocal analysis that have led to advances in understanding problems such as wave propagation, the Laplacian on asymptotically hyperbolic spaces and the meromorphic continuation of the dynamical zeta function for Anosov flows.

IL10.13

Geometric approaches to semilinear elliptic equations

Juncheng Wei
Chinese University of Hong Kong, Hong Kong and The University of British Columbia, Canada
wei@math.cuhk.edu.hk

2010 *Mathematics Subject Classification.* 35J61, 35B06, 35B25
Keywords. De Giorgi's Conjecture, minimal surfaces, Allen-Cahn equation, Nonlinear Schrodinger equation, gluing techniques

A fundamental problem in nonlinear PDE is the classification and construction of entire solutions of nonlinear elliptic equations in $\mathbb{R}^N$ such as $\Delta u + f(u) = 0$ in $\mathbb{R}^N$. This is the context of various classical results in the literature like the Gidas-Ni-Nirenberg theorems on radial symmetry, Liouville type theorems, or the achievements around De Giorgi's conjecture. In this paper we review some recent mathematical results on applying geometric approaches towards geometrization of solutions of entire solutions. We focus particularly on two prototype nonlinear elliptic equations: Allen-Cahn equation and nonlinear Schrödinger equation. For Allen-Cahn, we describe the De Giorgi conjecture and the connections with minimal surfaces as well as Toda systems. For nonlinear Schrödinger equation we are interested in new entire solutions with either finite energy or multiple ends. We discuss its surprising connection with the theory of Constant Mean Curvature (CMC) surfaces and Toda system. Applications to gauged Ginzburg-Landau equation as well as Chern-Simons-Higgs will be given.

IL10.9

Duality in Boltzmann equation and its applications

Shih-Hsien Yu
National University of Singapore, Singapore
matysh@nus.edu.sg

2010 *Mathematics Subject Classification.* 35L65, 35L67, 35Q67, 35E05, 82b40

Keywords. Boltzmann equation, Green's function, Shock Wave, Boundarly Layer, Stability

In this paper we will survey a quantitative and qualitative development on the Boltzmann equation. This development reveals the dual natures of the Boltzmann equation: The particlelike nature and the fluidlike nature. This dual nature property gives rise to the precise construction of the Green's function for Boltzmann equation around a global Maxwellian state. With the precise structure of the Green's function, one can implement the Green's function to study various problems such as invariant manifolds for the steady Boltzmann flows, time asymptotic nonlinear stability of Boltzmann shock layers and Boltzmann boundary layers, Riemann Problem, and bifurcation problem of boundary layer problem, etc.

11. Mathematical Physics

IL11.1

Three lives of the Gelfand-Zeitlin integrable system

Anton A. Alekseev
Université de Genève, Switzerland
Anton.Alekseev@unige.ch

2010 *Mathematics Subject Classification.* 53D17, 15B57
Keywords. Gelfand-Zeitlin integrable systems, Poisson structures, planar network

Gelfand-Zeiltin integrable systems were discovered by Guillemin and Sternberg in 1983, and they represent a standard reference point in the vast world of complete integrability. One of their characteristic features is that action variables satisfy the interlacing inequalities which govern eigenvalues of Hermitian matrices and their principal submatrices.

In the paper, we explain that besides the standard Linear Algebra interpretation there are two other, seemingly unrelated situations where interlacing inequalities and Gelfand-Zeitlin systems naturally arise. The first one is combinatorics of planar networks with Boltzmann weights on their edges. Surprizingly, it turns out that maximal weights of multi-paths in planar networks verify exactly the same inequalities as eigenvalues of Hermitian matrices. The second one is tropicalization of Poisson structures. We show that tropicalization of the canonical Poisson structure on the dual Poisson-Lie group $U^*(n)$ produces an integrable system isomorphic to the Gelfand-Zeitlin system. The link between the three topics comes from the synthesis of ideas of tropicalization, Total Positivity and Poisson-Lie groups.

As an application, we sketch a new symplectic proof of Horn inequalities for the spectrum of the sum of two Hermitian matrices with given eigenvalues.

IL11.2

Macdonald processes, quantum integrable systems and the Kardar-Parisi-Zhang universality class

Ivan Corwin
Clay Mathematics Institute, Columbia University and Institute Henri Poincare, United States of America
ivan.corwin@gmail.com

2010 *Mathematics Subject Classification.*
Keywords. Symmetric function theory, Quantum integrable systems, Kardar-Parisi-Zhang universality class, Non-equilibrium statistical mechanics

Integrable probability has emerged as an active area of research at the interface of probability/mathematical physics/statistical mechanics on the one hand, and representation theory/integrable systems on the other. Informally, integrable probabilistic systems have two properties: 1) It is possible to write down concise and exact formulas for expectations of a variety of interesting observables (or functions) of the system. 2) Asymptotics of the system and associated exact formulas provide access to exact descriptions of the properties and statistics of large universality classes and universal scaling limits for disordered systems. We focus here on examples of integrable probabilistic systems related to the Kardar-Parisi-Zhang (KPZ) universality class and explain how their integrability stems from connections with symmetric function theory and quantum integrable systems.

IL11.3

Liouville Quantum Gravity, KPZ and Schramm-Loewner Evolution

Bertrand Duplantier
Institut de Physique Theorique, France
bertrand.duplantier@cea.fr

2010 *Mathematics Subject Classification.* 81T40, 60J67

Keywords. Liouville quantum gravity, Gaussian free field, Knizhnik-Polyakov-Zamolodchikov relation, duality, Schramm-Loewner evolution, conformal welding

We describe a canonical model of random surfaces, Liouville quantum gravity, its relation to the Gaussian free field (GFF) and to the canonical model of conformally invariant random curves in the plane, the Schramm-Loewner evolution (SLE). The Liouville random measure is formally written as $\mu_\gamma(dz) = e^{\gamma h(z)}\, dz$, where dz is the planar Lebesgue measure, h an instance of the GFF, and $\gamma \in [0, 2)$. We outline a probabilistic and geometrical proof of the Knizhnik-Polyakov-Zamolodchikov (KPZ) relation between the scaling exponents of a fractal with respect to the Euclidean and Liouville measures, including the boundary geometry case. The Liouville quantum measure in the $\gamma = 2$ critical case is defined after a further logarithmic renormalization, yielding an atom-free measure satisfying the KPZ relation. When $\gamma > 2$, the measure is purely atomic, and is related to a dual quantum measure $\mu_{\gamma'}$ by $\gamma\gamma' = 4$. For $\gamma < 2$, the conformal welding of boundary arcs of a γ-Liouville quantum gravity surface (in a quantum boundary length-preserving way) produces an SLE_κ curve, with $\kappa = \gamma^2$. This allows one to develop a theory of quantum fractal measures on the SLE curve itself (consistent with the KPZ relation) and analyze their evolution under conformal welding maps.

IL11.11

An overview of the topological recursion

Bertrand Eynard
Institut de Physique Théorique, France
bertrand.eynard@cea.fr

2010 *Mathematics Subject Classification.* 14H70, 14H60, 14H50, 14H81

Keywords. Topological recursion, spectral curve, moduli spaces, mirror symmetry

We recall how computing large size asymptotics in random matrices, has allowed to discover some fascinating and ubiquitous geometric invariants. Specializations of this method recover many classical invariants, like Gromov–Witten invariants, or knot polynomials (Jones, HOMFLY,...). In this short overview we give some examples, give definitions, and review some properties and applications of the formalism.

IL11.5

Cluster varieties and integrable systems

Vladimir V. Fock
Université de Strasbourg et CNRS, France
fock@math.unistra.fr

2010 *Mathematics Subject Classification.* 14H70, 13F60, 70H06

Keywords. Integrable system, Cluster variety, Planar curve, Affine Lie group

In this lecture we present a combinatorial approach to integrable systems on affine Poisson-Lie groups using cluster technique and use it to study their properties such as discrete flows and explicit solutions.

IL11.6

Archimedian Langlands duality and exactly solvable quantum systems

Anton A. Gerasimov
Institute of Theoretical and Experimental Physics, Russian Federation
anton.a.gerasimov@gmail.com

2010 *Mathematics Subject Classification.* 11F66, 81R12, 81T4, 11F70

Keywords. Langlands duality, Baxter operator, L-factors, Toda chain, mirror symmetry

We demonstrate that Baxter operators for $\mathfrak{gl}_{\ell+1}$-Toda chains understood as elements of spherical Archimedean Hecke algebras provide a concise formulation of a special case of the local Archimedean Langlands correspondence. Categorification of the class one eigenfunctions of the q-deformed $\mathfrak{gl}_{\ell+1}$-Toda chain supplies a q-analog of the Shintani-Casselman-Shalika formula over non-Archimedean fields, thus revealing a q-version of the local Langlands correspondence. In the non-deformed case the q-analog of the formula turns into an expression of a matrix element of $GL_{\ell+1}(\mathbb{R})$ principal series spherical representation as the equivariant volume of an infinite-dimensional symplectic manifold. This provides another manifestation of the local Archimedean Langlands correspondence. Reformulation in terms of two-dimensional topological field theories allows identification of the considered instance of the Archimedean Langlands correspondence as mirror symmetry in two-dimensional quantum field theories.

IL11.10

Lyapunov functional approach and collective dynamics of some interacting many-body systems

Seung-Yeal Ha
Seoul National University, Republic of Korea
syha@snu.ac.kr

2010 *Mathematics Subject Classification.* 34H05, 35B35

Keywords. Complete synchronization, Cucker-Smale model, Lyapunov function, global flocking, Kuramoto model

A Lyapunov functional approach is a standard tool for studying the nonlinear stability of equilibria in the theory of dynamical systems. In this paper, we survey recent progress on the collective dynamics of interacting many-body systems and discuss how the Lyapunov functional approach can be used in the formation of collective motions, such as in flocking and the synchronization of many-body systems. We also propose some open questions in the mathematical theory of flocking and synchronization.

IL11.8

Fermionic spectra in integrable models

Rinat Kedem
University of Illinois at Urbana-Champaign, United States of America
rinat@illinois.edu

2010 *Mathematics Subject Classification.* 81R10, 82B23, 05E10

Keywords. Fermionic character formulas, Fusion products, discrete integrable systems

This is a brief review of several algebraic constructions related to generalized fermionic spectra, of the type which appear in integrable quantum spin chains and integrable quantum field theories. We discuss the connection between fermionic formulas for the graded dimensions of the spaces of conformal blocks of WZW theories, quantum cluster algebras, discrete integrable noncommutative evolutions and difference equations.

IL11.9

Deformed ensembles of random matrices

Sandrine Péché
Université Paris-Diderot, France
sandrine.peche@math.univ-paris-diderot.fr

2010 *Mathematics Subject Classification.* 60B20, 15B52, 62H20

Keywords. Probability, random matrices, separation of eigenvalues, universality

In this article, we review recent results in the study of asymptotic spectral properties of some perturbation of large random matrices. Deformed models have arisen in random matrix theory in Baik, J.; Ben Arous, G.; Péché, S. *Phase transition of the largest eigenvalue for nonnull complex sample covariance matrices.* Ann. Probab. **33** (2005), no. 5, 1643–1697. In this review, we consider additive or multiplicative deformations of standard Wigner or sample covariance matrices. We consider the phenomenon of separation of extreme eigenvalues and the question of universality of their asymptotic distribution for random matrices with a non necessarily Gaussian distribution.

IL11.7

Structure of the excitation spectrum for many-body quantum systems

Robert Seiringer
IST Austria, Austria
robert.seiringer@ist.ac.at

2010 *Mathematics Subject Classification.* 82B10, 82-06, 46N50

Keywords. Schr'odinger equation, quantum statistical mechanics, Bose-Einstein condensation, dilute Bose gas, superfluidity

Many questions concerning models in quantum mechanics require a detailed analysis of the spectrum of the corresponding Hamiltonian, a linear operator on a suitable Hilbert space. Of particular relevance for an understanding of the low-temperature properties of a system is the structure of the excitation spectrum, which is the part of the spectrum close to the spectral bottom. We present recent progress on this question for bosonic many-body quantum systems with weak two-body interactions. Such system are currently of great interest, due to their experimental realization in ultra-cold atomic gases. We investigate the accuracy of the Bogoliubov approximations, which predicts that the low-energy spectrum is made up of sums of elementary excitations, with linear dispersion law at low momentum. The latter property is crucial for the superfluid behavior the system.

IL11.4

Gauge theory angle at quantum integrability

Samson L. Shatashvili
Trinity College Dublin, Ireland
samson@maths.tcd.ie

2010 *Mathematics Subject Classification.* 79.0, 81E13, 81E30, 81E40, 81T30

Keywords. Gauge Theory, Supersymmetry, Quantum Integrability, TQFT, CFT

We review the relationship between supersymmetric gauge theories and quantum integrable systems. From the quantum integrability side this relation includes various spin chains, as well as many well-known quantum many body systems like elliptic Calogero-Moser system and generalisations. From the gauge theory side one has the supersymmetric gauge theory with four (and eight) supercharges in the space-time background which is a product of a d-dimensional torus, or a two dimensional cigar with Omega-deformation, and a flat space (with the total dimension of space-time being 2, 3, 4 or 5). The gauge theory perspective provides the exact energy spectrum of the corresponding quantum integrable system. Key notions, usually appearing in the topic of quantum integrability, such as Baxter equation, Yang-Yang function, Bethe equation, spectral curve, Yangian, quantum affine algebra, quantum elliptic algebra - all acquire meaning in these gauge theories.

IL11.12

Quantization of moduli spaces of flat connections and Liouville theory

Jörg Teschner
Universität Hamburg, Germany
joerg.teschner@desy.de

2010 *Mathematics Subject Classification.* 79.0

Keywords. Conformal Field Theory, Moduli spaces of flat connections, Hyperbolic geometry

We review known results on the relations between conformal field theory and the quantization of moduli spaces of flat $PSL(2, R)$-connections on Riemann surfaces.

12. Probability and Statistics

IL12.12

Operator limits of random matrices

Bálint Virág
University of Toronto, Canada
balint@math.toronto.edu

2010 *Mathematics Subject Classification.* 60B11, 62H12
Keywords. Random matrix, random operator

We present a brief introduction to the theory of operator limits of random matrices to non-experts. Several open problems and conjectures are given. Connections to statistics, integrable systems, orthogonal polynomials, and more, are discussed.

IL12.1

A short survey of Stein's method

Sourav Chatterjee
Stanford University, United States of America
souravc@stanford.edu

2010 *Mathematics Subject Classification.* 60F05, 60B10
Keywords. Stein's method, Normal approximation, Central limit theorem

Stein's method is a powerful technique for proving central limit theorems in probability theory when more straightforward approaches cannot be implemented easily. I will begin with a survey of the historical development of Stein's method and some recent advances. This will be followed by a description of a "general purpose" variant of Stein's method that may be called the generalized perturbative approach, and an application of this method to minimal spanning trees. I will conclude with the descriptions of some well known open problems that may possibly be solved by the perturbative approach or some other variant of Stein's method.

IL12.2

Criticality, universality, and isoradiality

Geoffrey R. Grimmett
University of Cambridge, United Kingdom
grg@statslab.cam.ac.uk

2010 *Mathematics Subject Classification.* 60K35, 82B20
Keywords. Percolation, random-cluster model, Ising/Potts models, critical point, universality, isoradial graph, critical exponent, star?triangle transformation, Yang?Baxter equation

Critical points and singularities are encountered in the study of critical phenomena in probability and physics. We present recent results concerning the values of such critical points and the nature of the singularities for two prominent probabilistic models, namely percolation and the more general random-cluster model. The main topic is the statement and proof of the criticality and universality of the canonical measure of bond percolation on isoradial graphs (due to the author and Ioan Manolescu). The key technique used in this work is the star-triangle

transformation, known also as the Yang–Baxter equation. The second topic reported here is the identification of the critical point of the random-cluster model on the square lattice (due to Beffara and Duminil-Copin), and of the criticality of the canonical measure of the random-cluster model with $q \geq 4$ on periodic isoradial graphs (by the same authors with Smirnov). The proof of universality for percolation is expected to extend to the random-cluster model on isoradial graphs.

IL12.6

Singular stochastic PDEs

Martin Hairer
The University of Warwick, United Kingdom
M.Hairer@Warwick.ac.uk

2010 *Mathematics Subject Classification.* 60H15, 81S20, 82C28
Keywords. Regularity structures, renormalisation, stochastic PDEs

We present a series of recent results on the well-posedness of very singular parabolic stochastic PDEs. These equations are such that the question of what it even means to be a solution is highly non-trivial. This problem can be addressed within the framework of the recently developed theory of "regularity structures", which allows to describe candidate solutions locally by a "jet", but where the usual Taylor polynomials are replaced by a sequence of custom-built objects. In order to illustrate the theory, we focus on the particular example of the KPZ equation.

IL12.9

The MM proximal distance algorithm

Kevin Keys[1] and Kenneth Lange[1,a*]
[1]University of California, Los Angeles, United States of America
[a]klange@ucla.edu

2010 *Mathematics Subject Classification.* 90C59, 65C60
Keywords. Majorization, convexity, exact penalty method, computational statistics

The MM principle is a device for creating optimization algorithms satisfying the ascent or descent property. The current survey emphasizes the role of the MM principle in nonlinear programming. For smooth functions, one can construct an adaptive interior point method based on scaled Bregmann barriers. This algorithm does not follow the central path. For convex programming subject to nonsmooth constraints, one can combine an exact penalty method with distance majorization to create versatile algorithms that are effective even in discrete optimization. These proximal distance algorithms are highly modular and reduce to set projections and proximal mappings, both very well-understood techniques in optimization. We illustrate the possibilities in linear programming, binary piecewise-linear programming, nonnegative quadratic programming, ℓ_0 regression, matrix completion, and inverse sparse covariance estimation.

IL12.3

Anomalous random walks and diffusions: From fractals to random media

Takashi Kumagai
Kyoto University, Japan
kumagai@kurims.kyoto-u.ac.jp

2010 *Mathematics Subject Classification.* 60J45, 05C81, 60K37

Keywords. Fractals, Heat kernel estimates, Percolation, Random media, Sub-diffusivity

We present results concerning the behavior of random walks and diffusions on disordered media. Examples treated include fractals and various models of random graphs, such as percolation clusters, trees generated by branching processes, Erdős-Rényi random graphs and uniform spanning trees. As a consequence of the inhomogeneity of the underlying spaces, we observe anomalous behavior of the corresponding random walks and diffusions. In this regard, our main interests are in estimating the long time behavior of the heat kernel and in obtaining a scaling limit of the random walk. We will overview the research in these areas chronologically, and describe how the techniques have developed from those introduced for exactly self-similar fractals to the more robust arguments required for random graphs.

IL12.5

Heat flows, geometric and functional inequalities

Michel Ledoux
Université de Toulouse, France
ledoux@math.univ-toulouse.fr

2010 *Mathematics Subject Classification.* 35K05, 39B62, 47D07, 53C21, 60J60

Keywords. Heat flow, Markov diffusion semigroup, geometric and functional inequality, curvature bound, gradient bound

Heat flow and semigroup interpolations have developed over the years as a major tool for proving geometric and functional inequalities. Main illustrations presented here range over logarithmic Sobolev inequalities, heat kernel bounds, isoperimetric-type comparison theorems, Brascamp-Lieb inequalities and noise stability. Transportation cost inequalities from optimal mass transport are also part of the picture as consequences of new Harnack-type inequalities. The geometric analysis involves Ricci curvature lower bounds via, as a cornerstone, equivalent gradient bounds on the diffusion semigroups. Most of the results presented here are joint with D. Bakry.

IL12.4

Determinantal probability: Basic properties and conjectures

Russell Lyons
Indiana University, United States of America
rdlyons@indiana.edu

2010 *Mathematics Subject Classification.* 60K99, 60G55, 42C30, 37A15, 37A35

Keywords. Random matrices, negative association, point processes, orthogonal polynomials, completeness

(1) For each subset A of the circle with measure m, there is a sequence of integers of Beurling-Malliavin density m such that the set of corresponding complex exponentials is complete for $L^2(A)$.

(2) Given an infinite graph, simple random walk on each tree in the wired uniform spanning forest is a.s. recurrent.

(3) Let Z be the set of zeroes of a random Gaussian power series in the unit disk. Then a.s., the only function in the Bergman space that vanishes on Z is the zero function.

(4) In our talk, we explain a theorem that has (1) and (2) as corollaries. We also describe a conjectural extension that has (3) (which is not known) as a corollary. All these depend on determinantal probability measures.

IL12.7

Rough paths, signatures and the modelling of functions on streams

Terry Lyons
University of Oxford, United Kingdom
terry.lyons@oxford-man.ox.ac.uk

2010 *Mathematics Subject Classification.*

Keywords. Rough paths, Regularity Structures, Machine Learning, Numerical Approximation of Parabolic PDE, Shuffle Product and Tensor Algebra

Rough path theory is focused on capturing and making precise the interactions between highly oscillatory and non-linear systems. The techniques draw particularly on the analysis of LC Young and the geometric algebra of KT Chen. The concepts and theorems, and the uniform estimates, have found widespread application; the first applications gave simplified proofs of basic questions from the large deviation theory and substantially extending Ito?s theory of SDEs; the recent applications contribute to (Graham) automated recognition of Chinese handwriting and (Hairer) formulation of appropriate SPDEs to model randomly evolving interfaces. At the heart of the mathematics is the challenge of describing a smooth but potentially highly oscillatory and vector valued path xt parsimoniously so as to effectively predict the response of a nonlinear system such as $dy_t = f(y_t)dx_t, y_0 = a$. The signature is a homomorphism from the monoid of paths into the grouplike elements of a closed tensor algebra. It provides a graduated summary of the path x. Hambly and Lyons have shown that this non-commutative transform is faithful for paths of bounded variation up to appropriate null modifications. Among paths of bounded variation with given Signature there is always a unique shortest representative. These algebra predicated summaries or features of a path are at the heart of the definition of a rough path; locally they remove the need to look at the fine structure of the path. Taylor?s theorem explains how any smooth function can, locally, be expressed as a linear combination of certain special functions (monomials based at that point). Coordinate Iterated integrals form a more subtle algebra of features that can describe a stream or path in an analogous way; they allow a definition of rough path and a natural linear ?basis??for functions on streams that can be used for machine learning.

IL12.8

Variational formulas for directed polymer and percolation models

Timo Seppäläinen
University of Wisconsin-Madison, United States of America
seppalai@math.wisc.edu

2010 *Mathematics Subject Classification.* 60K35, 60K37, 82B41

Keywords. Corner growth model, directed polymer, directed percolation, log-gamma polymer

Explicit formulas for subadditive limits of polymer and percolation models in probability and statistical mechanics have been difficult to find. We describe variational formulas for these limits and connections with other features of the models such as Busemann functions and Kardar-Parisi-Zhang (KPZ) fluctuation exponents.

IL12.11

Criticality and Phase Transitions: five favorite pieces

Vladas Sidoravicius
IMPA, Brazil
v.sidoravicius@gmail.com

2010 *Mathematics Subject Classification.* 60K35

Keywords. Phase transition, Criticality, Percolation, Non-equilibrium phase transition

We present few recent results concerning the behavior of classical equilibrium and non-equilibrium systems at criticality. Five topics are discussed: a) continuity of the phase transition for Bernoulli percolation, Ising and Potts models; b) geometry of critical percolation clusters in context of self-destructive percolation; c) non-equilibrium phase transitions, critical behavior of conservative lattice gasses; d) dynamic phase transitions in KPZ type growth systems in presence of columnar defect and solution of slow bond problem; e) solution of Coffman-Gilbert conjecture.

IL12.10

Aggregation and minimax optinality in high-dimensional estimation

Alexandre B. Tsybakov
CREST-ENSAE, France
alexandre.tsybakov@upmc.fr

2010 *Mathematics Subject Classification.* 62G05, 62J07

Keywords. High-dimensional model, aggregation, sparsity, oracle inequality, minimax estimation

Aggregation is a popular technique in statistics and machine learning. Given a collection of estimators, the problem of linear, convex or model selection type aggregation consists in constructing a new estimator, called the aggregate, which is nearly as good as the best among them

(or nearly as good as their best linear or convex combination), with respect to a given risk criterion. When the underlying model is sparse, which means that it is well approximated by a linear combination of a small number of functions in the dictionary, the aggregation techniques turn out to be very useful in taking advantage of sparsity. On the other hand, aggregation is a general technique of producing adaptive nonparametric estimators, which is more powerful than the classical methods since it allows one to combine estimators of different nature.Aggregates are usually constructed by mixing the initial estimators or functions of the dictionary with data-dependent weights that can be defined is several possible ways. Important example is given by aggregates with exponential weights. They satisfy sharp oracle inequalities that allow one to treat in a unified way three different problems: Adaptive nonparametric estimation, aggregation and sparse estimation.

IL12.13

Constrained forms of statistical minimax: Computation, communication, and privacy

Martin J. Wainwright
UC Berkeley, United States of America
wainwrig@stat.berkeley.edu

2010 *Mathematics Subject Classification.* 62Cxx, 68W40

Keywords. Statistical minimax, Information theory, Metric entropy, Differential privacy, Computational complexity

A fundamental quantity in statistical decision theory is the notion of the minimax risk associated with an estimation problem. It is based on a saddlepoint problem, in which nature plays the role of adversary in choosing the underlying problem instance, and the statistician seeks an estimator with good properties uniformly over a class of problem instances. We argue that in many modern estimation problems arising in the mathematical sciences, the classical notion of minimax risk suffers from a significant deficiency: to wit, it allows for all possible estimators, including those with prohibitive computational costs, unmanageable storage requirements, or other undesirable properties. Accordingly, we introduce some refinements of minimax risk based on imposing additional constraints on the sets of possible estimators. We illustrate this notion of constrained statistical minimax via three vignettes, based on restrictions involving computation, communication, and privacy, respectively.

13. Combinatorics

IL13.2

Coloring graphs with forbidden induced subgraphs

Maria Chudnovsky
Columbia University, United States of America
mchudnov@columbia.edu

2010 *Mathematics Subject Classification.* 05C15, 05C85
Keywords. Graph coloring, induced subgraphs, coloring algorithms

Since graph-coloring is an NP-complete problem in general, it is natural to ask how the complexity changes if the input graph is known not to contain a certain induced subgraph H. Results of Kamínski and Lozin, Holyer, and Levin and Galil imply that the problem remains NP-complete, unless H is the disjoint union of paths. Recently, the question of coloring graphs that do not contain certain induced paths has received considerable attention. Only one case of that problem remains open for k-coloring when $k \geq 4$, and that is the case of 4-coloring graphs with no induced 6-vertex path. However, little is known for 3-coloring. In this paper we survey known results on the topic, and discuss recent developments.

IL13.1

Combinatorial theorems relative to a random set

David Conlon
University of Oxford, United Kingdom
david.conlon@maths.ox.ac.uk

2010 *Mathematics Subject Classification.* 05C80, 05C35
Keywords. Extremal combinatorics, Random graphs

We describe recent advances in the study of random analogues of combinatorial theorems.

IL13.10

The graph regularity method: variants, applications, and alternative methods

Jacob Fox
Massachusetts Institute of Technology, United States of America
fox@math.mit.edu

2010 *Mathematics Subject Classification.* 05C35, 05C65, 05D10, 05D40
Keywords. Regularity lemma, Ramsey theory, extremal combinatorics, probabilistic methods

Szemerédi's regularity lemma is one of the most powerful tools in graph theory, with many applications in combinatorics, number theory, discrete geometry, and theoretical computer science. Roughly speaking, it says that every large graph can be partitioned into a small number of parts such that the bipartite subgraph between almost all pairs of parts is random-like. Several variants of the regularity lemma have since been established with many further applications. This survey discusses recent progress in understanding the quantitative aspects of

these lemmas and their applications, as well as recent progress in developing a sparse regularity method.

IL13.3

Positional games

Michael Krivelevich
Tel Aviv University, Israel
krivelev@post.tau.ac.il

2010 *Mathematics Subject Classification.* 05C57, 91A46, 05C80, 05D05, 05D10

Keywords. Positional games, Ramsey theory, extremal set theory, probabilistic intuition

Positional games are a branch of combinatorics, researching a variety of two-player games, ranging from popular recreational games such as Tic-Tac-Toe and Hex, to purely abstract games played on graphs and hypergraphs. It is closely connected to many other combinatorial disciplines such as Ramsey theory, extremal graph and set theory, probabilistic combinatorics, and to computer science. We survey the basic notions of the field, its approaches and tools, as well as numerous recent advances, standing open problems and promising research directions.

IL13.4

Hamilton cycles in graphs and hypergraphs: an extremal perspective

Daniela Kühn[1,a*] and Deryk Osthus[1,b]
[1]University of Birmingham, United Kingdom
[a]d.kuhn@bham.ac.uk
[b]d.osthus@bham.ac.uk

2010 *Mathematics Subject Classification.* 05C45, 05C35, 05C65, 05C20

Keywords. Hamilton cycles, Hamilton decompositions, factorizations, hypergraphs, graph packings and coverings

As one of the most fundamental and well-known NP-complete problems, the Hamilton cycle problem has been the subject of intensive research. Recent developments in the area have highlighted the crucial role played by the notions of expansion and quasi-randomness. These concepts and other recent techniques have led to the solution of several long-standing problems in the area. New aspects have also emerged, such as resilience, robustness and the study of Hamilton cycles in hypergraphs. We survey these developments and highlight open problems, with an emphasis on extremal and probabilistic approaches.

IL13.5

Random planar graphs and beyond

Marc Noy
Universitat Politécnica de Catalunya, Spain
marc.noy@upc.edu

2010 *Mathematics Subject Classification.* 05A16, 05B80

Keywords. Asymptotic enumeration, Random graphs, Planar graphs, Graph minors

We survey several results on the enumeration of planar graphs and on properties of random planar graphs. This includes basic parameters, such as the number of edges and the number of connected components, and extremal parameters such as the size of the largest component, the diameter and the maximum degree. We discuss extensions to graphs on surfaces and to classes of graphs closed under minors. Analytic methods provide very precise results for random planar graphs. The results for general minor-closed classes are less precise but hold with wider generality.

IL13.6

The Gelfand-Tsetlin graph and Markov processes

Grigori Olshanski
Institute for Information Transmission Problems, Russian Federation
olsh2007@gmail.com

2010 *Mathematics Subject Classification.* 05E05, 05E10, 60J27, 60J35

Keywords. Asymptotic representation theory, representation ring, Gelfand-Tsetlin graph, Feller Markov processes, infinitesimal generators

The goal of the paper is to describe new connections between representation theory and algebraic combinatorics on one side, and probability theory on the other side.

The central result is a construction, by essentially algebraic tools, of a family of Markov processes. The common state space of these processes is an infinite dimensional (but locally compact) space Ω. It arises in representation theory as the space of indecomposable characters of the infinite-dimensional unitary group $U(\infty)$.

Alternatively, Ω can be defined in combinatorial terms as the boundary of the Gelfand-Tsetlin graph — an infinite graded graph that encodes the classical branching rule for characters of the compact unitary groups $U(N)$.

We also discuss two other topics concerning the Gelfand-Tsetlin graph:

(1) Computation of the number of trapezoidal Gelfand-Tsetlin schemes (one could also say, the number of integral points in a truncated Gelfand-Tsetlin polytope). The formula we obtain is well suited for asymptotic analysis.

(2) A degeneration procedure relating the Gelfand-Tsetlin graph to the Young graph by means of a new combinatorial object, the Young bouquet.

At the end we discuss a few related works and further developments.

IL13.7

Geometric intersection patterns and the theory of topological graphs

János Pach
EPFL, Switzerland and Rényi Institute, Hungary
pach@cims.nyu.edu

2010 *Mathematics Subject Classification.* 05C35, 05C62, 52C10

Keywords. Intersection graph, topological graph, Ramsey theory, separator, partial order

The intersection graph of a set system S is a graph on the vertex set S, in which two vertices are connected by an edge if and only if the corresponding sets have nonempty intersection. It was shown by Tietze (1905) that every finite graph is the intersection graph of 3-dimensional convex polytopes. The analogous statement is false in any fixed dimension if the polytopes are allowed to have only a bounded number of faces or are replaced by simple geometric objects that can be described in terms of a bounded number of real parameters. Intersection graphs of various classes of geometric objects, even in the plane, have interesting structural and extremal properties.

We survey problems and results on geometric intersection graphs and, more generally, intersection patterns. Many of the questions discussed were originally raised by Berge, Erdos, Grunbaum, Hadwiger, Turan, and others in the context of classical topology, graph theory, and combinatorics (related, e.g., to Helly?s theorem, Ramsey theory, perfect graphs). The rapid development of computational geometry and graph drawing algorithms in the last couple of decades gave further impetus to research in this field. A topological graph is a graph drawn in the plane so that its vertices are represented by points and its edges by possibly intersecting simple continuous curves connecting the corresponding point pairs. We give applications of the results concerning intersection patterns in the theory of topological graphs.

IL13.9

The determinism of randomness and its use in combinatorics

Angelika Steger
ETH Zürich, Switzerland
steger@inf.ethz.ch

2010 *Mathematics Subject Classification.* 05C80, 05A16

Keywords. Random graph theory, probabilistic methods, extremal graphs, average case analysis, percolation theory

Many areas of science, most notably statistical physics, rely on the use of probability theory to explain key phenomena. The aim of this article is to explore the role of probability in combinatorics. More precisely, our aim is to cover a wide range of topics that illustrate the various roles that probability plays within combinatorics: from just providing intuition for deterministic statements, like Szemerédi?s regularity lemma or the recent container theorems, over statements about random graphs with structural side constraints and average case analysis of combinatorial algorithms, all the way to neuroscience.

IL13.8

Combinatorial problems in random matrix theory

Van H. Vu
Yale University, United States of America
van.vu@yale.edu

2010 *Mathematics Subject Classification.* 05D40, 15B52, 60C05

Keywords. Random discrete matrices, singularity, rank, determinant, inverse theorems

In this survey, we discuss several combinatorial problems in Random Matrix theory. We will present the current status of these problems, together with some key ideas and open questions.

14. Mathematical Aspects of Computer Science

IL14.1

Sum-of-squares proofs and the quest toward optimal algorithms

Boaz Barak[1,a*] and David Steurer[2]
[1]Microsoft Research, United States of America
[2]Cornell University, United States of America
[a]b@boazbarak.org

2010 *Mathematics Subject Classification.* 68Q25, 90C22
Keywords. Sum of squares, Semidefinite programming, Unique Games Conjecture, Small set expansion

In order to obtain the best-known guarantees, algorithms are traditionally tailored to the particular problem we want to solve. Two recent developments, the Unique Games Conjecture (UGC) and the Sum-of-Squares (SOS) method, surprisingly suggest that this tailoring is not necessary and that a single efficient algorithm could achieve best possible guarantees for a wide range of different problems.

The Unique Games Conjecture (UGC) is a tantalizing conjecture in computational complexity, which, if true, will shed light on the complexity of a great many problems. In particular this conjecture predicts that a single concrete algorithm provides optimal guarantees among all efficient algorithms for a large class of computational problems.

The Sum-of-Squares (SOS) method is a general approach for solving systems of polynomial constraints. This approach is studied in several scientific disciplines, including real algebraic geometry, proof complexity, control theory, and mathematical programming, and has found applications in fields as diverse as quantum information theory, formal verification, game theory and many others.

We survey some connections that were recently uncovered between the Unique Games Conjecture and the Sum-of-Squares method. In particular, we discuss new tools to rigorously bound the running time of the SOS method for obtaining approximate solutions to hard optimization problems, and how these tools give the potential for the sum-of-squares method to provide new guarantees for many problems of interest, and possibly to even refute the UGC.

IL14.2

Interactive information and coding theory

Mark Braverman
Princeton University, United States of America
mbraverm@cs.princeton.edu

2010 *Mathematics Subject Classification.* 94A15, 68Q99
Keywords. Coding theory, communication complexity, information complexity, interactive computation

We give a high-level overview of recent developments in interactive information and coding theory. These include developments involving interactive noiseless coding and interactive error-correction.

The overview is primarily focused on developments related to complexity-theoretic applications, although the broader context and agenda are also set out. As the present paper is an extended abstract, the vast majority of proofs and technical details are omitted, and can be found in the respective publications and preprints.

IL14.3

Counting constraint satisfaction problems

Andrei A. Bulatov
Simon Fraser University, Canada
abulatov@sfu.ca

2010 *Mathematics Subject Classification.* 68Q25, 68Q17
Keywords. Constraint satisfaction problem, counting, complexity, partition function, homomorphism

Counting constraint satisfaction problems (CSPs) originate from two very different areas: statistical physics, where partition functions appearing in "spin-glass" models have been studied since the beginning of the last century, and counting combinatorial problems formally introduced by Valiant in the late 70s. In spite such a long history, the systematic study of the general counting CSP started less than 15 years ago. In this short survey we review recent results on counting CSPs.

IL14.4

Flows, cuts and integral routing in graphs - an approximation algorithmist's perspective

Julia Chuzhoy
Toyota Technological Institute at Chicago, United States of America
cjulia@ttic.edu

2010 *Mathematics Subject Classification.* 68Q25, 68Q17, 68R05, 68R10
Keywords. Maximum flow, Minimum cut, Network routing, Approximation algorithms, Graph theory

Flow, cut and integral graph routing problems are among the most extensively studied in Operations Research, Optimization, Graph Theory and Computer Science. We survey known algorithmic results for these problems, including classical results and more recent developments, and discuss the major remaining open problems, with an emphasis on approximation algorithms.

IL14.5

Computing on the edge of chaos: structure and randomness in encrypted computation

Craig Gentry
IBM T.J. Watson Research Center, United States of America
craigbgentry@gmail.com

2010 *Mathematics Subject Classification.* 68Qxx, 68P25

Keywords. Cryptography, complexity theory, homomorphic encryption, software obfuscation, learning with errors (LWE)

This survey, aimed mainly at mathematicians rather than practitioners, covers recent developments in homomorphic encryption (computing on encrypted data) and program obfuscation (generating encrypted but functional programs). Current schemes for encrypted computation all use essentially the same "noisy" approach: they encrypt via a noisy encoding of the message, they decrypt using an "approximate" ring homomorphism, and in between they employ techniques to carefully control the noise as computations are performed. This noisy approach uses a delicate balance between structure and randomness: structure that allows correct computation despite the randomness of the encryption, and randomness that maintains privacy against the adversary despite the structure. While the noisy approach "works", we need new techniques and insights, both to improve efficiency and to better understand encrypted computation conceptually.

IL14.6

Social choice, computational complexity, Gaussian geometry, and Boolean functions

Ryan O'Donnell
Carnegie Mellon University, United States of America
odonnell@cs.cmu.edu

2010 *Mathematics Subject Classification.* 68Q87, 94C10, 60G15

Keywords. Social choice, analysis of Boolean functions, computational complexity, Gaussian geometry, isoperimetry

We describe a web of connections between the following topics: the mathematical theory of voting and social choice; the computational complexity of the Maximum Cut problem; the Gaussian Isoperimetric Inequality and Borell's generalization thereof; the Hypercontractive Inequality of Bonami; and, the analysis of Boolean functions. A major theme is the technique of reducing inequalities about Gaussian functions to inequalities about Boolean functions $f : \{-1,1\}^n \to \{-1,1\}$, and then using induction on n to further reduce to inequalities about functions $f : \{-1,1\} \to \{-1,1\}$. We especially highlight De, Mossel, and Neeman's recent use of this technique to prove the Majority Is Stablest Theorem and Borell's Isoperimetric Inequality simultaneously.

IL14.7

Algorithms for circuits and circuits for algorithms: Connecting the tractable and intractable

Ryan Williams
Stanford University, United States of America
rrwilliams@gmail.com

2010 *Mathematics Subject Classification.* 68Q17, 68Q25

Keywords. Circuit complexity, algorithm analysis, satisfiability, lower bounds, derandomization

The title of this paper highlights an emerging duality between two basic topics in algorithms and complexity theory.

"Algorithms for circuits" refers to the design of algorithms which can analyze finite logical circuits or Boolean functions as input, checking a simple property about the complexity of the underlying function. For instance, an algorithm determining if a given logical circuit C has an input that makes C output true, would solve the NP-complete Circuit-SAT problem. Such an algorithm is unlikely to run in polynomial time, but could possibly be more efficient than exhaustively trying all possible inputs to the circuit.

"Circuits for algorithms" refers to the modeling of complex uniform algorithms with simple Boolean circuit families, or proving that such modeling is impossible. For example, can every exponential-time algorithm be simulated using Boolean circuit families of only polynomial size? It is widely conjectured that the answer is no, but the present mathematical tools available are still too crude to resolve this kind of separation problem.

This paper surveys these two generic subjects and the connections that have been developed between them, focusing on connections between non-trivial circuit-analysis algorithms and proofs of circuit complexity lower bounds.

IL14.8

Codes with local decoding procedures

Sergey Yekhanin
Microsoft Research, United States of America
yekhanin@microsoft.com

2010 *Mathematics Subject Classification.* 94B05, 94B35, 68R05, 68P20

Keywords. Error correcting codes, Locally decodable codes, Multiplicity codes, Matching vectors codes, Maximally recoverable codes

Error correcting codes allow senders to add redundancy to messages, encoding bit strings representing messages into longer bit strings called codewords, in a way that the message can still be recovered even if a fraction of the codeword bits are corrupted. In certain settings however the receiver might not be interested in recovering all the message, but rather seek to quickly recover just a few coordinates of it. Codes that allow one to recover individual message coordinates extremely fast (locally), from accessing just a small number of carefully chosen coordinates of a corrupted codeword are said to admit a local decoding procedure. Such codes have recently played an important role in several areas of theoretical computer science and have also been used in practice to provide reliability in large distributed storage systems. We survey what is known about these codes.

15. Numerical Analysis and Scientific Computing

IL15.1

On a class of high order schemes for hyperbolic problems

Rémi Abgrall
University of Zurich, Switzerland
remi.abgrall@math.uzh.ch

2010 *Mathematics Subject Classification.* 65, 76

Keywords. Numerical approximation of hyperbolic problems, Non oscillatory schemes, Unstructured meshes, High order methods

This paper provides a review about a family of non oscillatory and parameter free finite element type methods for advection-diffusion problems. Due to space limitation, only the scalar hyperbolic problem is considered. We also show that this class of schemes can be interpreted as finite volume schemes with multidimensional fluxes.

IL15.4

Spline differential forms

Annalisa Buffa
Istituto di Matematica Applicata e Tecnologie Informatiche "E. Magenes", C.N.R., Italy
annalisa.buffa@imati.cnr.it

2010 *Mathematics Subject Classification.* 65N30, 65D07

Keywords. Numerical analysis, spline theory, discretization of partial differential equations

We introduce spline discretization of differential forms and study their properties. We analyse their geometric and topological structure, as related to the connectivity of the underlying mesh, we present degrees of freedom and we construct commuting projection operators, with optimal stability and approximation properties.

IL15.3

Multiscale model reduction with generalized multiscale finite element methods

Yalchin Efendiev
Texas A&M University, United States of America
efendiev@math.tamu.edu

2010 *Mathematics Subject Classification.* 65N99, 65N30

Keywords. Multiscale, finite element, model reduction, homogenization, porous media

Many application problems have multiscale nature. Due to disparity of scales, the simulations of these problems are prohibitively expensive. Some types of upscaling or model reduction techniques are needed to solve many multiscale problems. In this talk, we discuss a few known techniques that are used for problems with scale separation and focus on Generalized Multiscale Finite Element Method (GMsFEM) that has been recently proposed for solving problems with non-separable scales and high contrast. The main objective of the method is to provide local reduced-order approximations for linear and nonlinear PDEs via multiscale

spaces on a coarse computational grid. In the talk, we briefly discuss some main concepts of constructing multiscale spaces and applications of GMsFEMs.

IL15.2

Discontinuous Galerkin method for time-dependent convection dominated partial differential equations

Chi-Wang Shu
Brown University, United States of America
shu@dam.brown.edu

2010 *Mathematics Subject Classification.* 65M60, 65M20, 65M12, 65M15

Keywords. Discontinuous Galerkin method, time-dependent convection dominated partial differential equations, hyperbolic equations, convection-diffusion equations, stability

In this lecture we give an introduction to discontinuous Galerkin (DG) methods for solving time-dependent convection dominated partial differential equations (PDEs). DG methods form a class of finite element methods. Differently from classical finite element methods, which are built upon spaces containing continuous, piecewise polynomial functions, DG methods are built upon function spaces containing piecewise polynomials (or other simple functions) which are allowed to be completely discontinuous across element interfaces. Using finite element terminologies, DG methods are the most extreme case of nonconforming finite element methods. DG methods are most natural and most successful for solving hyperbolic conservation laws which have generic discontinuous solutions. Moreover, in recent years stable and convergent DG methods have also been designed for convection dominated PDEs containing higher order spatial derivatives, such as convection diffusion equations and KdV equations. We will emphasize the guiding principles for the design and analysis, and recent development and applications of the DG methods for solving time-dependent convection dominated PDEs.

IL15.5

Singular stochastic computational models, stochastic analysis, PDE analysis, and numerics

Denis Talay
Inria Sophia Antipolis, France
denis.talay@inria.fr

2010 *Mathematics Subject Classification.* 60H30, 60H35, 65C05, 65C30, 60C35

Keywords. Stochastic numerics, Applications of stochastic analysis to partial differential equations and numerical analysis

Stochastic computational models are aimed to simulate complex physical or biological phenomena and to approximate (deterministic) macroscopic physical quantities by means of probabilistic numerical methods. By nature, they often involve singularities and are subject to the curse of dimensionality. Their efficient and accurate simulation is still an open question in many aspects.

The aim of this lecture is to review some recent developments concerning the numerical

approximation of singular stochastic dynamics, and to show novel issues in stochastic analysis and PDE analysis they lead to.

IL15.6

A review on subspace methods for nonlinear optimization

Ya-xiang Yuan
Chinese Academy of Sciences, China
yyx@lsec.cc.ac.cn

2010 *Mathematics Subject Classification.* 65K05, 90C30
Keywords. Numerical methods, nonlinear optimization, subspace techniques, subproblems

In this paper, we review various subspace techniques that have been used in constructing numerical methods for solving nonlinear optimization problems. As large scale optimization problems are attracting more and more attention in recent years, subspace methods are getting more and more important since they do not require solving large scale subproblems in each iteration. The essential parts of a subspace method are how to construct subproblems defined in lower dimensional subspaces and how to choose the subspaces in which the subproblems are defined. Various subspace methods for unconstrained optimization, constrained optimization, nonlinear equations and nonlinear least squares, and matrix optimization problems are given respectively, and different proposals are made on how to choose the subspaces.

16. Control Theory and Optimization

IL16.1

Recent results around the diameter of polyhedra

Friedrich Eisenbrand
EPFL, Switzerland
friedrich.eisenbrand@epfl.ch

2010 *Mathematics Subject Classification.* 52B11, 52B55

Keywords. Convex Geometry, Linear Programming, Polyhedra, Algorithms

The diameter of a polyhedron P is the largest distance of a pair of vertices in the edge-graph of P. The question whether the diameter of a polyhedron can be bounded by a polynomial in the dimension and number of facets of P remains one of most important open problems in convex geometry.

In the last three years, there was an accelerated interest in this famous open problem which has lead to many interesting results and techniques, also due to a celebrated breakthrough of Santos disproving the Hirsch conjecture. Here, I want to describe a subset of these recent results and describe some open problems.

IL16.5

Optimization over polynomials: selected topics

Monique Laurent
Centrum Wiskunde & Informatica and Tilburg University, Netherlands
monique@cwi.nl

2010 *Mathematics Subject Classification.* 90C22, 90C27, 90C30, 44A60, 13J30

Keywords. Positive polynomial, sum of squares, moment problem, combinatorial optimization, semidefinite optimization

Minimizing a polynomial function over a region defined by polynomial inequalities models broad classes of hard problems from combinatorics, geometry and optimization. New algorithmic approaches have emerged recently for computing the global minimum, by combining tools from real algebra (sums of squares of polynomials) and functional analysis (moments of measures) with semidefinite optimization. Sums of squares are used to certify positive polynomials, combining an old idea of Hilbert with the recent algorithmic insight that they can be checked efficiently with semidefinite optimization. The dual approach revisits the classical moment problem and leads to algorithmic methods for checking optimality of semidefinite relaxations and extracting global minimizers. We review some selected features of this general methodology, illustrate how it applies to some combinatorial graph problems, and discuss links with other relaxation methods.

IL16.3

Nonsmooth optimization: conditioning, convergence, and semi-algebraic models

Adrian Lewis
Cornell University, United States of America
adrian.lewis@cornell.edu

2010 *Mathematics Subject Classification.* 90C31, 49K40, 65K10, 14P10, 93D20

Keywords. Variational analysis, nonsmooth optimization, inverse function, alternating projections, metric regularity, semi-algebraic, convergence rate, condition number, normal cone, transversality, quasi-Newton, eigenvalue optimization, identifiable manifold

Variational analysis has come of age. Long an elegant theoretical toolkit for variational mathematics and nonsmooth optimization, it now increasingly underpins the study of algorithms, and a rich interplay with semi-algebraic geometry illuminates its generic applicability. As an example, alternating projections - a rudimentary but enduring algorithm for exploring the intersection of two arbitrary closed sets - concisely illustrates several far-reaching and interdependent variational ideas. A transversality measure, intuitively an angle and generically nonzero, controls several key properties: the method's linear convergence rate, a posteriori error bounds, sensitivity to data perturbations, and robustness relative to problem description. These linked ideas emerge in a wide variety of computational problems. Optimization in particular is rich in examples that depend, around critical points, on "active" manifolds of nearby approximately critical points. Such manifolds, central to classical theoretical and computational optimization, exist generically in the semi-algebraic case. We discuss examples from eigenvalue optimization and stable polynomials in control systems, and a prox-linear algorithm for large-scale composite optimization applications such as machine learning.

IL16.4

Carleman estimates, results on control and stabilization for partial differential equations

Luc Robbiano
Université de Versailles Saint-Quentin-en-Yvelines, France
luc.robbiano@uvsq.fr

2010 *Mathematics Subject Classification.* 35A02, 35Q93, 93B05, 93D15

Keywords. Carleman estimates, control, null control, stabilization, exterior problem

In this survey we give some results based on Carleman estimates. We recall the classical uniqueness result based on interior Carleman estimate. We give Carleman estimate up the boundary useful for the applications. The main applications are, approximative control for wave equation, null control for heat equation, stabilization for wave equation for an interior damping or for a boundary damping and local energy decay for wave equation in exterior domain.

IL16.2

Models and feedback stabilization of open quantum systems

Pierre Rouchon
Mines Paris Tech, France
pierre.rouchon@mines-paristech.fr

2010 *Mathematics Subject Classification.* 93B52, 93D15, 81V10, 81P15

Keywords. Markov model, open quantum system, quantum filtering, quantum feedback, quantum master equations

At the quantum level, feedback-loops have to take into account measurement back-action. We present here the structure of the Markovian models including such back-action and sketch two stabilization methods: measurement-based feedback where an open quantum system is stabilized by a classical controller; coherent or autonomous feedback where a quantum system is stabilized by a quantum controller with decoherence (reservoir engineering). We begin to explain these models and methods for the photon box experiments realized in the group of Serge Haroche (Nobel Prize 2012). We present then these models and methods for general open quantum systems.

IL16.6

Time-inconsistent optimal control problems

Jiongmin Yong
University of Central Florida, United States of America
jiongmin.yong@ucf.edu

2010 *Mathematics Subject Classification.* 93E20, 49L20, 49N05, 49N70

Keywords. Stochastic optimal control, time inconsistency, equilibrium solution, Hamilton-Jacobi-Bellman equation, differential games

An optimal control problem is time-consistent if for any initial pair of time and state, whenever there exists an optimal control, it will stay optimal thereafter. In real world, however, such kind of time-consistency is hardly true, mainly due to the time-inconsistency of decision maker's time-preference and/or risk-preference. In another word, most optimal control problems, if not all, are not time-consistent, or time-inconsistent. In this paper, some general time-inconsistent optimal control problems are formulated for stochastic differential equations. Recent works of the author concerning the (time-consistent) equilibrium solutions to the time-inconsistent problems are surveyed.

17. Mathematics in Science and Technology

IL17.1

Mathematical Models and Numerical Methods for Bose-Einstein Condensation

Weizhu Bao
National University of Singapore, Singapore
matbaowz@nus.edu.sg

2010 *Mathematics Subject Classification.* 35Q55, 70F10

Keywords. Bose-Einstein condensation, Gross-Pitaevskii equation, nonlinear Schrödinger equation, ground state, dynamics

The achievement of Bose-Einstein condensation (BEC) in ultracold vapors of alkali atoms has given enormous impulse to the theoretical and experimental study of dilute atomic gases in condensed quantum states inside magnetic traps and optical lattices. This article offers a short survey on mathematical models and theories as well as numerical methods for BEC based on the mean field theory. We start with the Gross-Pitaevskii equation (GPE) in three dimensions (3D) for modeling one-component BEC of the weakly interacting bosons, scale it to obtain a three-parameter model and show how to reduce it to two dimensions (2D) and one dimension (1D) GPEs in certain limiting regimes. Mathematical theories and numerical methods for ground states and dynamics of BEC are provided. Extensions to GPE with an angular momentum rotation term for a rotating BEC, to GPE with long-range anisotropic dipole-dipole interaction for a dipolar BEC and to coupled GPEs for spin-orbit coupled BECs are discussed. Finally, some conclusions are drawn and future research perspectives are discussed.

IL17.4

Discrete-to-continuum variational methods for lattice systems

Andrea Braides
Universitá di Roma Tor Vergata, Italy
braides@mat.uniroma2.it

2010 *Mathematics Subject Classification.* 49J45, 35B27, 35Q70, 49D50, 49F22

Keywords. Discrete systems, Variational methods, Homogenization, Optimal design, Variational motion

I review some recent results regarding the description of the behaviour of energy-driven discrete systems, and more precisely lattice systems, through the construction of approximate continuous problems. On one hand methods of weak convergence, homogenization, integral representation and gradient flow dynamics already used for continuum problems have been adapted to the discrete setting, on the other hand the new discrete dimension has brought new phenomena, novel problems and interesting results. I will limit my description to systems with interfacial energies, but focus on methods that can be adapted to a multi-scale analysis

IL17.3

Mathematical models and numerical methods for electronic structure calculation

Eric Cancès
École des Ponts ParisTech, France
cances@cermics.enpc.fr

2010 *Mathematics Subject Classification.* 81Q05, 35A15

Keywords. Schrödinger equation, Variational methods, Quantum chemistry, Solid state physics, Materials science

This contribution provides a pedagogical introduction for mathematicians to the field of electronic structure calculation. The N-body electronic Schrödinger equation and the main methods to approximate the solutions to this equation (wavefunction methods, density functional theory, quantum Monte Carlo) are presented. The numerical simulation of the resulting models, the construction of electronic structure models for systems with infinitely many electrons (perfect crystals, crystals with local defects, disordered materials) by means of thermodynamic limits, and the modeling and simulation of molecules interacting with complex environments, are discussed.

IL17.5

A mathematical perspective of image denoising

Miguel Colom[1] , Gabriele Facciolo[2] , Marc Lebrun[2] , Jean-Michel Morel[2,a*] , Nicola Pierazzo[2] , Martin Rais[1] , and Yi-Qing Wang[2]
[1]Universitat Illes Balears, Spain
[2]Ecole Normale Supérieure de Cachan, France
[a]morel@cmla.ens-cachan.fr

2010 *Mathematics Subject Classification.* 62H35, 68U10, 94A08

Keywords. Image denoising, Fourier transform, Wiener estimate, wavelet threshold, discrete cosine transform, oracle estimate, Bayes formula, neighborhood filters, nonlocal methods, neural networks, blind denoising

Digital images are matrices of regularly spaced samples, the pixels, each containing a photon count. Each pixel thus contains a random sample of a Poisson variable. Its mean would be the ideal image value at this pixel. It follows that all images are random discrete processes and therefore “noisy”. Ever since digital images exist, numerical methods have been proposed to recover the ideal mean from its random observed value. This problem is obviously ill posed and makes sense only if there is an underlying image model. Inventing or learning from data a consistent mathematically image model is the core of the problem. Images being 2D projections of our complex surrounding visual world, this is a challenging problem, which is nevertheless beginning to find simple but mathematically innovative answers.

We shall distinguish four classes of denoising principles, relying on functional or stochastic image models. We show that each of these principles can be summarized in a single formula. In addition these principles can be combined efficiently to cope with the full image complexity. This explains their immediate industrial impact. All current cameras and imag-

ing devices rely directly on the simple formulas explained here. In the past ten years the image quality delivered to users has increased fast thanks to this exemplary mathematical modeling.

As an illustration of the universality and simplicity reached by the theory, most image denoising algorithms discussed in this paper can be tested directly on any digital image at *Image Processing On Line*, http://www.ipol.im/. In this web journal, each paper contains a complete algorithmic description, the corresponding source code, and can be run online on arbitrary images.

IL17.2

Sparse Analysis

Anna C. Gilbert
University of Michigan, United States of America
annacg@umich.edu

2010 *Mathematics Subject Classification.* 42, 68

Keywords. Sparse approximation, compressive sensing, sublinear algorithms, compression, sparse signal recovery

The goal of this lecture is to give you an introduction to the mathematics, algorithms, and applications in the field of sparse analysis, including sparse approximation and compressive sensing. Both of these problems contain a wealth of challenging algorithmic problems, novel uses of existing mathematical techniques, as well as mathematical innovations. Coupled with these theoretical challenges are practical engineering questions that both support and motivate the mathematical innovations.

The fundamental mathematical problem is that of solving an under-determined linear system. Despite learning in high school algebra that such problems are “impossible” to solve, mathematicians, computer scientists, and engineers attempt to do so in a myriad of fields, applications, and settings. This problem arises in signal and image compression, theoretical computer science, algorithms for massive, streaming data sets, high-throughput biological screens, and in the design of analog-to-digital converters.

IL17.6

Scaling in kinetic mean-field models for coarsening phenomena

Barbara Niethammer
Universität Bonn, Germany
niethammer@iam.uni-bonn.de

2010 *Mathematics Subject Classification.* 82C26, 35C06

Keywords. Coarsening, Ostwald Ripening, Grain Growth, scaling hypothesis

We consider two paradigms of coarsening systems in materials science, Ostwald Ripening and Grain Growth. Experimental observations suggest that for large times such systems evolve in a universal statistically self-similar fashion. One approach to capture this behaviour is to utilize kinetic mean-field models for the particle size distributions. We review recent progress in the derivation and the analysis of such equations for our two model examples.

IL17.7

Computing global invariant manifolds: techniques and applications

Hinke M. Osinga
The University of Auckland, New Zealand
h.m.osinga@auckland.ac.nz

2010 *Mathematics Subject Classification.* 37C10, 37D10, 37C70, 65L10, 65P30

Keywords. Dynamical systems, invariant manifold, boundary value problem, continuation techniques

Global invariant manifolds play an important role in organising the behaviour of a dynamical system. Together with equilibria and periodic orbits, they form the so-called skeleton of the dynamics and offer geometric insight into how observed behaviour arises. In most cases, it is impossible to find invariant manifolds explicitly and numerical methods must be used to find accurate approximations. Developing such computational techniques is a challenge on its own and, to this date, the focus has primarily been on computing two-dimensional manifolds. Nevertheless, these computational efforts offer new insights that go far beyond a confirmation of the known theory. Furthermore, global invariant manifolds in dynamical systems theory not only explain asymptotic behaviour, but more recent developments show that they are equally useful for explaining short-term transient dynamics. This paper presents an overview of these more recent developments, in terms of novel computational methods, as well as applications that have stimulated recent advances in the field and highlighted the need for new mathematical theory.

IL17.8

Numerical approximation of variational inequalities arising in elastoplasticity

B. Daya Reddy
University of Cape Town, South Africa
daya.reddy@uct.ac.za

2010 *Mathematics Subject Classification.* 65M60, 74C05, 65M15, 65N30

Keywords. Elastoplasticity, variational inequalities, finite elements, algorithms, predictor-corrector schemes

Mathematical models of many classes of nonsmoooth problems in mechanics take the form of variational inequalities. Elastoplasticity, which is a theory of solids that exhibit path-dependent and irreversible behaviour, yields a variational inequality that is not of standard elliptic or parabolic type. Properties of the corresponding abstract problem are reviewed, as are the conditions under which fully discrete approximations converge. A solution algorithm, motivated by the predictor-corrector algorithms that are common in elastoplastic problems, is constructed for the abstract problem and shown to converge.

IL17.9

Uncertainty Quantification in Bayesian inversion

Andrew M. Stuart
The University of Warwick, United Kingdom
a.m.stuart@warwick.ac.uk

2010 *Mathematics Subject Classification.* 35R30, 62C10

Keywords. Inverse problems, Bayesian inversion, Uncertainty quantification, Monte Carlo methods, Stochastic partial differential equations

Probabilistic thinking is of growing importance in many areas of mathematics. This paper highlights the beautiful mathematical framework, coupled with practical algorithms, which results from thinking probabilistically about inverse problems arising in partial differential equations.

Many inverse problems in the physical sciences require the determination of an unknown field from a finite set of indirect measurements. Examples include oceanography, oil recovery, water resource management and weather forecasting. In the Bayesian approach to these problems, the unknown and the data are modelled as a jointly varying random variable, typically linked through solution of a partial differential equation, and the solution of the inverse problem is the distribution of the unknown given the data.

This approach provides a natural way to provide estimates of the unknown field, together with a quantification of the uncertainty associated with the estimate. It is hence a useful practical modelling tool. However it also provides a very elegant mathematical framework for inverse problems: whilst the classical approach to inverse problems leads to ill-posedness, the Bayesian approach leads to a natural well-posedness and stability theory. Furthermore this framework provides a way of deriving and developing algorithms which are well-suited to the formidable computational challenges which arise from the conjunction of approximations arising from the numerical analysis of partial differential equations, together with approximations of central limit theorem type arising from sampling of measures.

IL17.10

Stochastic modeling and methods in optimal portfolio construction

Thaleia Zariphopoulou
The University of Texas at Austin, United States of America
zariphop@math.utexas.edu

2010 *Mathematics Subject Classification.* 97M30, 91G80

Keywords. Expected utility, forward investment performance, duality, robustness, stochastic PDE

Optimal portfolio construction is one of the most fundamental problems in financial mathematics. The foundations of investment theory are discussed together with modeling issues and various methods for the analysis of the associated stochastic optimization problems. Among others, the classical expected utility and its robust extension are presented as well as the recently developed approach of forward investment performance. The mathematical tools come from stochastic optimization for controlled diffusions, duality and stochastic partial differential equations. Connections between the academic research and the investment practice are

also discussed and, in particular, the challenges of reconciling normative and descriptive approaches.

18. Mathematics Education and Popularization of Mathematics

IL18.1

The internet and the popularization of mathematics

Étienne Ghys
CNRS, UMPA École Normale Supérieure de Lyon, France
etienne.ghys@ens-lyon.fr

2010 *Mathematics Subject Classification.* 00A09, 97A80, 97A40
Keywords. Popularization of mathematics, Internet

In this paper, "popularization of mathematics" is understood as the attempt to share some of the current mathematical research activity with the general public. I would like to focus on the internet as a powerful tool to achieve this goal. I report on three personal experiences: the making of two animation films available on the web, the participation to a web-journal aimed at a wide audience, and the filming of a 15 minute video clip.

IL18.2

Teaching and learning "What is Mathematics"

Andreas Loos[1] and Günter M. Ziegler[1,a*]
[1]Freie Universität Berlin, Germany
[a]ziegler@math.fu-berlin.de

2010 *Mathematics Subject Classification.* 97D30, 00A05, 01A80, 97D20
Keywords. "What is Mathematics?", the image/the images of mathematics, Klein's "double discontinuity", teaching mathematics/telling stories about mathematics, the "Panorama of Mathematics" project

"What is Mathematics?" (with a question mark!) is the title of a famous book by Courant and Robbins, first published in 1941, which does not answer the question. The question is, however, essential: The public image of the subject (of the science, and of the profession) is not only relevant for the support and funding it can get, but it is also crucial for the talent it manages to attract — and thus ultimately determines what mathematics can achieve, as a science, as a part of human culture, but also as a substantial component of economy and technology.

In this lecture we thus

- discuss the image of mathematics (where "image" might be taken literally!),
- sketch a multi-facetted answer to the question "What is Mathematics?,"
- stress the importance of learning "What is Mathematics" in view of Klein's "double discontinuity" in mathematics teacher education,
- present the "Panorama project" as our response to this challenge,
- stress the importance of *telling stories* in addition to *teaching* mathematics, and finally
- suggest that the mathematics curricula at schools and at universities should correspondingly have space and time for at least three different subjects called Mathematics.

19. History of Mathematics

IL19.1

Knowledge and Power: A Social history of the transmission of mathematics between China and Europe during the Kangxi reign (1662-1722)

Qi Han
Chinese Academy of Sciences, China
qihan63@hotmail.com

2010 *Mathematics Subject Classification.*

Keywords. Chinese mathematics, Jesuits, Kangxi Emperor, Leibniz, transmission

In the last few decades much research has been devoted to the interaction of European and Chinese mathematics in the seventeenth and eighteenth centuries. Scholars have begun to consider social and political factors in their studies of Chinese mathematics. This approach, however desirable, needs more systematic exploration. Drawing on research findings in social and political history, I will analyse why the Kangxi Emperor (1654-1722) began to be interested in European mathematics and how he used his newly acquired mathematical knowledge as a tool to control and impress Chinese official scholars and so consolidate his power. In addition, I will point out the reasons why he changed his attitude toward Western learning and established an Academy of Mathematics in 1713. Then I explore how European mathematical books were introduced and circulated in the Kangxi reign (1662-1722). Further I discuss why the Kangxi Emperor became interested in traditional Chinese mathematics. Finally, using both Chinese and European sources, I discuss the study of The Book of Changes (易经) at the imperial court and its link to the French Jesuit Joachim Bouvet (1656-1730) and the German philosopher Leibniz.

IL19.2

One hundred years after the Great War (1914–2014) - A century of breakdowns, resumptions and fundamental changes in international mathematical communication

Reinhard Siegmund-Schultze
University of Agder, Norway
reinhard.siegmund-schultze@uia.no

2010 *Mathematics Subject Classification.* 01A60, 01A61, 01A80

Keywords. International mathematical communication, World War I and II, emigration of mathematicians

The paper describes and analyzes changing political, social and institutional conditions for international mathematical communication during the last one hundred years. The focus is on the Western Hemisphere and on relatively peaceful times between and after the two wars. Topics include the boycott against German and Austrian science, Rockefeller support for the internationalization of mathematics, the mass exodus of mathematicians from Europe in the 1930s, the resumption of mathematical contacts after WWII, the growing awareness of mathematics in the Soviet Union, and the emigration of Russian scholars to the West before and

after the Fall of the Iron Curtain. Some emphasis is put on the barriers of language and culture between European, American and Russian mathematics and on the influence of Bourbaki during various periods. Several decisive events from the history of the ICM and the IMU are mentioned for their bearing on international communication.

IL19.3

Mathematics of engineers: Elements for a new history of numerical analysis

Dominique Tournès
University of La Reunion, France
dominique.tournes@univ-reunion.fr

2010 *Mathematics Subject Classification.* 65-03, 01A55, 01A60

Keywords. Mathematics of engineers, Numerical analysis, Nomography, Ballistics, Differential equations

The historiography of numerical analysis is still relatively poor. It does not take sufficient account of numerical and graphical methods created, used and taught by military and civil engineers in response to their specific needs, which are not always the same as those of mathematicians, astronomers and physicists. This paper presents some recent historical research that shows the interest it would be to examine more closely the mathematical practices of engineers and their interactions with other professional communities to better define the context of the emergence of numerical analysis as an autonomous discipline in the late 19th century.

Panel Discussions

ICM P-2

Mathematics is everywhere

Eduardo Colli[1] , Fidel Nemenzo[2] , Konrad Polthier[3] , and Christiane Rousseau[4,a*]
[1]Universidade de Sao Paulo, Brazil
[2]University of Philippines Diliman, Philippines
[3]Frei Universitat Berlin, Germany
[4]University of Montreal, Canada
[a]rousseac@dms.umontreal.ca

2010 *Mathematics Subject Classification.* 00A05, 00B10

Keywords. Popularization of mathematics, Mathematics a living discipline, Mathematics is everywhere

In the first part of the panel, the four panelists will briefly discuss and illustrate with practical examples what can be put under this title, what are the messages that can be passed to the public, and how to pass these messages. To most mathematicians, it seems obvious that mathematics is everywhere, and a living discipline within science and technology. Yet, how many of them are able to convey the message? And, when most people look around, they do not see mathematics, they do not know about the mathematics underlying the technology, they know very little about the role of mathematics in the scientific venture. Can we help building a powerful message? Can we unite forces for better passing it?

The second half of the panel will consist in a debate with the room.

ICM P-7

Mathematics communication for the future: a vision slam and special exhibition visit

Carla Cederbaum[1,a] , Gert-Martin Greuel[2,b*] , David Grünberg[3,c] , Hyungju Park[4,d] , and Cedric Villani[5,e]
[1]MFO and University of Tübingen, Germany
[2]University of Kaiserslautern, Germany
[3]International School of Lausanne, Switzerland
[4]National Institute of Mathematical Sciences, Republic of Korea
[5]Institut Henri Poincaré, France
[a]cederbaum@mfo.de
[b]greuel@mathematik.uni-kl.de
[c]david.grunberg@isl.ch
[d]alanpark@postech.ac.kr
[e]villani@ihp.fr

2010 *Mathematics Subject Classification.* 00A09, 97A80: Popularization of mathematics, 00A66: Mathematics and visual arts, visualization, 00A08, 97A20: Recreational mathematics

Keywords. Mathematics communication, vision slam, exhibition visit, interactive mathematics

Interactive mathematics exhibitions, mathematical theorems in books for a broad public, open source and collaborative networks and platforms, school children developing mathematical museum exhibits, ….

Mathematics communication is as dynamic as never before. New formats are being developed. New forms of collaborations between mathematicians, teachers, artists, and the public are being established. Platforms for mathematics communicators are connecting research institutes and individuals.

In this “vision slam”, active players in the field of mathematics communication will present their visions for future mathematics communication. You will hear different perspectives from the renowned mathematicians Cedric Villani and ICM chair Hyungju Park to the young researcher Carla Cederbaum and the teacher David Grünberg. The format of this session is simple: each speaker has 10 minutes to catch the audience‘s attention and transmit novel thoughts and ideas. Be curious. The talks will be inspiring and non-formal. Please be there at the beginning of the slam at 3 pm.

The panel is held under the umbrella of IMAGINARY, thus called “IMAGINARY panel”. IMAGINARY is a project by the Mathematisches Forschungsinstitut Oberwolfach (MFO), supported by the Klaus Tschira Foundation. IMAGINARY shaped collaborative and interactive mathematics communication over the last years on an international level. The project will be briefly introduced by Gert-Martin Greuel, former director of the MFO. IMAGINARY ideas will also be present in most of the vision slam talks.

After the vision slam, everybody is invited to visit the NIMS IMAGINARY exhibition shown at the conference. Mathematicians, contributors, developers, and team members of the local exhibition present in Seoul will guide you and talk about details and insights into the software programs, images, films, or 3d sculptures shown at the exhibition. This special exhibition tour will start at 4 pm and is open ended.

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